



UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
International General Certificate of Secondary Education

CANDIDATE NAME

CENTER NUMBER

CANDIDATE NUMBER



ADDITIONAL MATHEMATICS (US) **0459/01**
Paper 1 **May/June 2013**
2 hours

Candidates answer on the Question Paper
Additional Materials: Electronic calculator
List of formulas and statistical tables (MF25)

READ THESE INSTRUCTIONS FIRST

Write your Center number, candidate number, and name on the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of points is given in parentheses [] at the end of each question or part question.
The total number of points for this paper is 80.

This document consists of **16** printed pages.

- 1 A circle is given by the equation

$$x^2 + y^2 - 8x + 6y + 8 = 0.$$

Find the radius and the coordinates of the center of the circle.

[4]

- 2 Show that $\tan \theta \left(\frac{1}{1 - \cos \theta} - \frac{1}{1 + \cos \theta} \right)$ can be written as $\frac{k}{\sin \theta}$ and find the value of k .

3 A sequence of terms is defined recursively by

$$f(0) = 3, \quad f(1) = 5, \quad f(n+1) = kf(n) - f(n-1) \text{ for } n \geq 1.$$

Given that $f(3) = 9$, find the possible values of k .

[5]

4 (i) Show that $\left(\frac{x^{\frac{1}{4}} - x^{-\frac{1}{4}}}{x^{\frac{1}{4}}}\right)^2 = 1 - 2x^{-\frac{1}{2}} + x^{-1}$.

(ii) Hence solve $(1 - 2x^{-\frac{1}{2}} + x^{-1})^{\frac{1}{2}} x^{\frac{1}{2}} = 5$. [3]

5 The position vectors of the points A and B , relative to an origin O , are $5\mathbf{i} + 7\mathbf{j}$ and $9\mathbf{i} + 9\mathbf{j}$ respectively. The position vector of the point C , relative to O , is $k\mathbf{i} + 19\mathbf{j}$, where k is a positive constant.

(i) Find the value of k for which the length of AC is 20 units. [3]

(ii) Find the value of k for which ABC is a straight line. [3]

- 6 From a random sample of the heights of 100 female college students, unbiased estimates of the population mean and standard deviation were found to be 172 cm and 6 cm respectively. Assuming a normal distribution, showing the estimated population percentages in each of the following classes.

Height, h cm

$$h \leq 160$$

$$160 < h \leq 165$$

$$165 < h \leq 170$$

$$170 < h \leq 175$$

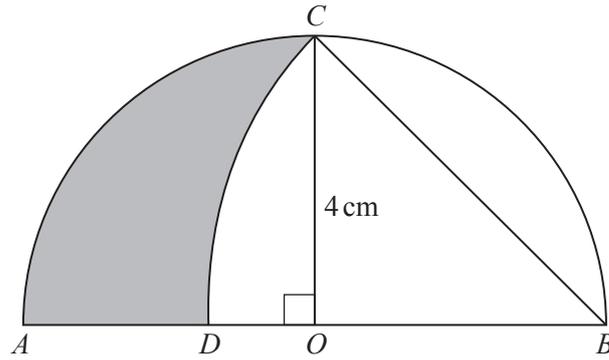
$$175 < h \leq 180$$

$$180 < h \leq 185$$

$$185 < h$$

[6]

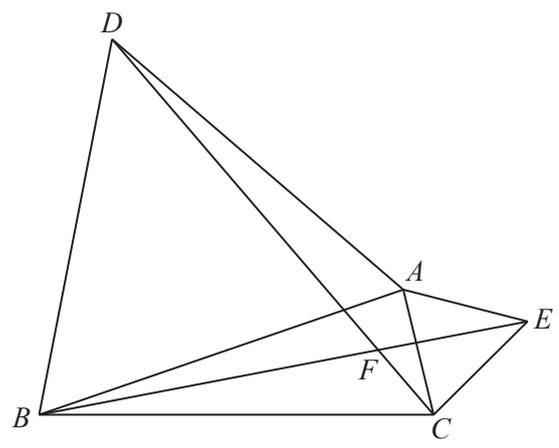
7



The diagram shows a semicircle of radius 4 cm with center O . The radius OC is perpendicular to the diameter AB . An arc of a circle is drawn with center B and radius BC . The arc meets AB at D .

(i) Show that $BD = 4\sqrt{2}\text{ cm}$ and find the length of the arc CD . [4]

(ii) Find the area of the shaded region. [4]



The diagram shows a triangle ABC together with equilateral triangles ADB and AEC . The lines BE and CD intersect at F . Prove that

(i) triangles ADC and ABE are congruent, [3]

(ii) angle $BFC = 120^\circ$. [4]

- 9 Given the points $A(2, 3)$, $B(4, 0)$ and $C(6, 3.5)$,
- (i) find the equation of the perpendicular bisector of AB ,

- (ii) verify that C lies on the perpendicular bisector of AB .

[1]

Given also that the point D is such that the mid-point of CD is also the mid-point of AB ,

(iii) find the coordinates of D ,

(iv) explain why the quadrilateral $ACBD$ is a rhombus.

[1]

10 Given that $\mathbf{A} = \begin{pmatrix} 3 & 2 \\ -1 & 1 \end{pmatrix}$, use the inverse matrix of \mathbf{A} to

(i) solve the system of equations

$$\begin{aligned} 2y + 3x - 4 &= 0, \\ y - x - 7 &= 0, \end{aligned}$$

[5]

- (ii) find the matrix \mathbf{B} such that $(\mathbf{B} + \mathbf{I})\mathbf{A} = \begin{pmatrix} 5 & 5 \\ 8 & 7 \end{pmatrix}$.

11 The polynomial $f(x) = 2x^3 + ax^2 + bx + 15$ has $x + 3$ as a factor. When $f(x)$ is divided by $x - 3$ the remainder is -60 .

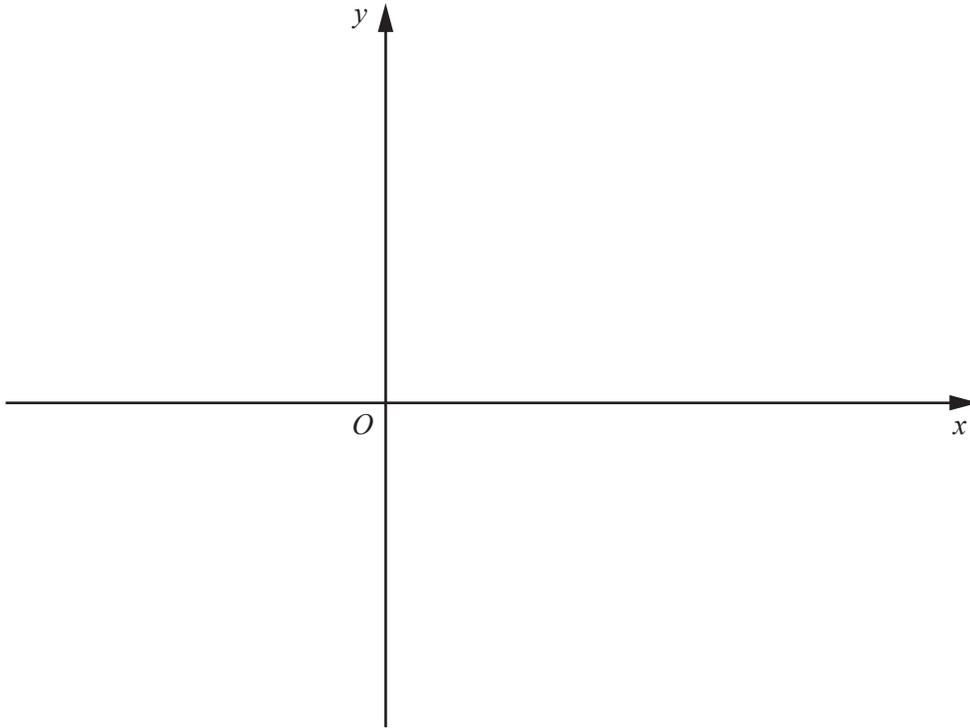
(i) Show that $a = -5$ and find the value of b .

[5]

(ii) Solve $f(x) = 0$.

[4]

- (iii) Sketch the graph of $y = f(x)$, stating the coordinates of the points where the graph crosses the axes.



Question 12 is printed on the next page.

12 Two events, A and B , are such that $P(A) = 0.6$, $P(B) = 0.3$ and $P(B|A) = 0.4$. Calculate the probability that

(i) either A or B occurs, but not both,

[5]

(ii) neither A nor B occurs.

[2]

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