UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

FURTHER MATHEMATICS **9231/02**

Paper 2

May/June 2006

3 hours

Additional Materials: Answer Booklet/Paper
Graph paper
List of Formulae (MF10)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
Where a numerical value is necessary, take the acceleration due to gravity to be $10\,\mathrm{m\,s^{-2}}$.
The use of a calculator is expected, where appropriate.
Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
At the end of the examination, fasten all your work securely together.

This document consists of **5** printed pages and **3** blank pages.

UNIVERSITY *of* CAMBRIDGE International Examinations

1 A ball drops vertically onto a smooth plane inclined to the horizontal at an angle α. It hits the plane with speed $8\,\mathrm{m\,s^{-1}}$ and rebounds horizontally. The coefficient of restitution between the ball and the plane is $\frac{1}{3}$. Find the value of α and the speed with which the ball rebounds. [5]

2 A particle rests on a horizontal platform which oscillates vertically in simple harmonic motion with period 2.5 seconds. During the motion the particle does not leave the platform. Show that the greatest possible value of the amplitude of the motion is approximately 1.58 m. [5]

In the case when the amplitude is exactly 1.58 m the lowest position of the particle is at the point A. Find the time for the particle to move directly to B, where $AB = 2$ m. [2]

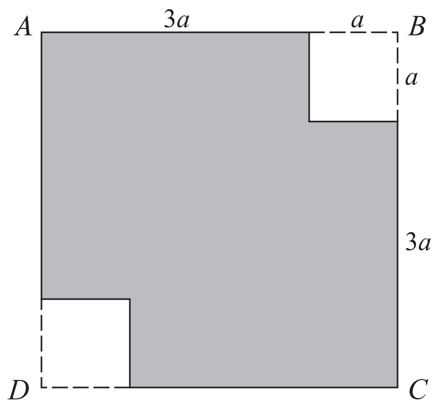
3 Two smooth uniform spheres A and B, of equal radii but of masses 20 grams and 10 grams respectively, lie at rest on a smooth horizontal table. They are projected directly towards each other, A with speed $5\,\mathrm{m\,s^{-1}}$ and B with speed $3\,\mathrm{m\,s^{-1}}$. The coefficient of restitution between the spheres is e.

(i) Find, in terms of e, the velocities of A and B after they collide. [5]

(ii) Find the set of values of e for which the directions of motion of both spheres are reversed in the collision. [2]

(iii) For the case $e = \frac{1}{5}$, find the magnitude of the impulse that B exerts on A as a result of the collision. [2]

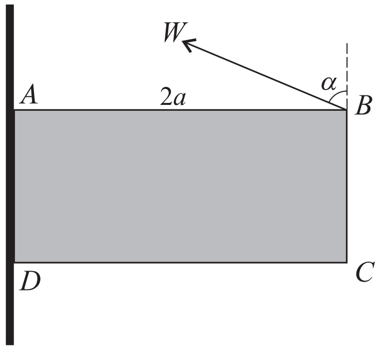
4



A uniform square plate $ABCD$, of side $4a$ and mass M, has squares of side a cut from the corners at B and D (see diagram). Show that the moment of inertia of the resulting lamina about an axis through A and perpendicular to the plane of the lamina is $\frac{109}{12}Ma^2$. [8]

The lamina is free to rotate in a vertical plane about this axis and is released from rest with AC horizontal. Find the initial acceleration of C. [3]

5



A uniform rectangular lamina $ABCD$ has $AB = 2a$ and weight W. It is supported with AD in contact with a rough vertical wall. The lamina is in a vertical plane perpendicular to the wall and is held in equilibrium by a force of magnitude W acting at B. This force acts at an angle α to the vertical and in the plane of the lamina (see diagram). The coefficient of friction between the lamina and the wall is μ. Show that $\mu \geqslant \tan\frac{1}{2}\alpha$. [7]

The normal component of the force of the wall on the lamina acts at the point E on AD, where $AE = x$. Show that

$$x = \frac{a(1 - 2\cos\alpha)}{\sin\alpha}.$$ [2]

Find the set of values of α for which equilibrium is possible. [2]

6 A car makes a regular journey of 40 km, but owing to varying traffic the time for the journey varies. The time, T hours, may be modelled by a random variable with probability density function given by

$$f(t) = \begin{cases} \dfrac{1}{2t^3} & t \geqslant \frac{1}{2}, \\ 0 & \text{otherwise.} \end{cases}$$

The average speed for the journey is A km h^{-1}.

(i) Find the distribution function of T and show that the distribution function of A is given, for $0 \leqslant a \leqslant 80$, by

$$G(a) = \frac{a^2}{6400}.$$ [5]

(ii) Find the probability that the average speed for a randomly chosen journey exceeds 60 km h^{-1}. [2]

7 In an investigation into the effect of environment on patient recovery, one group of 12 randomly chosen patients who had received the same surgery were each given a room with a pleasant view. A second group of 10 randomly chosen patients who had received the same surgery were each given a room facing a brick wall. The recovery times, in days, for the patients were recorded. The results, x and y, for the larger and smaller groups respectively, are summarised as follows.

$$\Sigma x = 93.5 \quad \Sigma x^2 = 734.0 \quad \Sigma y = 91.0 \quad \Sigma y^2 = 833.9$$

The population recovery times for the two groups have equal variance. Find an unbiased estimate of this variance. [2]

Stating any assumption that you need to make, calculate a 95% confidence interval for the difference $\mu_y - \mu_x$ between the population means. [4]

State, giving a reason, whether the interval supports a researcher's claim that patients given a room with a pleasant view recover on average one day earlier than patients given a room facing a brick wall. [1]

8 The useful life of a particular type of drill depends on the rate of revolution of the drill. Ten drills are tested, using 5 different rates of revolution, and the useful lives are recorded. The results, x thousand revolutions per minute and y hours, are given in the table.

x	1.5	1.5	2.0	2.0	2.5	2.5	3.0	3.0	3.5	3.5
y	5.2	5.8	6.1	4.9	4.3	3.9	3.8	2.6	1.4	1.7

$$[\Sigma x = 25,\ \Sigma y = 39.7,\ \Sigma x^2 = 67.5,\ \Sigma xy = 89.05.]$$

(i) Find the equation of the regression line of y on x in the form $y = a + bx$. [3]

The equation of the regression line of x on y is $x = 4.18 - 0.424y$, where the coefficients are given correct to 3 significant figures.

(ii) Use the regression coefficients of the two equations to find the product moment correlation coefficient. [2]

(iii) Estimate the useful life of a drill which rotates at 2250 revolutions per minute, and state how you decided which equation to use. [2]

(iv) Comment on the reliability of your answer to part **(iii)**. [1]

9 The interval, X metres, between consecutive minor faults on a roll of cloth has a negative exponential distribution whose probability density function is given by

$$\mathrm{f}(x) = \begin{cases} \lambda e^{-\lambda x} & x \geqslant 0, \\ 0 & \text{otherwise,} \end{cases}$$

where λ is a positive constant. It is given that $\mathrm{P}(X < 1) = 2\mathrm{P}(X > 2)$. Find the mean of the distribution. [6]

Repeated independent observations of X are taken, and the Nth observation is the first for which $X > 2$. Find $\mathrm{P}(N > 5)$. [3]

10 Torch batteries of a particular brand are sold in packs of 3. A store receives a large delivery of packs. The manager selects a random sample of 60 packs and tests all of the batteries. The numbers of faulty batteries in the packs are summarised in the following table.

Number faulty	0	1	2	3
Number of packs	28	28	4	0

(i) State one assumption about the batteries in a pack which you would need to make for a binomial distribution to be appropriate. [1]

(ii) Fit a binomial distribution to the data and carry out a goodness of fit test at the 10% significance level. [8]

(iii) 10 packs are selected at random from the delivery. Estimate the probability that at least two of these packs contain 3 faulty batteries. [3]

11 Answer only **one** of the following two alternatives.

EITHER

A particle P is suspended from a fixed point O by a light inextensible string of length a. When hanging at rest under gravity at A it is given a horizontal velocity u. The particle moves freely in a vertical circle and the string slackens when OP makes an angle of $\frac{1}{3}\pi$ with the upward vertical.

(i) Find u in terms of a and g. [6]

(ii) Verify that, when the string next becomes taut, the particle is at A. [6]

(iii) What is the speed of the particle just before it reaches A? [2]

OR

A random sample of university students consisted of 25 males and 15 females. Each was asked if there were areas of the campus where they were afraid to walk at night. 16 of the 40 students said that they were afraid. It is required to carry out a test at the 5% significance level of whether gender and fear are independent. The number of male students who said that they were afraid is denoted by n. Find the set of values of n for which it would be accepted that the two attributes are not independent. [11]

In another university, a random sample of 50 students was asked the same question. In this sample there were 32 males and 18 females. 8 of the males and 4 of the females said that they were afraid. Comment on the use, for this set of data, of a test similar to that used above. [3]

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