UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

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FURTHER MATHEMATICS **9231/01**

Paper 1 **October/November 2008**

3 hours

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF10)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a calculator is expected, where appropriate.
Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

UNIVERSITY of CAMBRIDGE International Examinations

[Turn over

1 The curve C is defined parametrically by

$$x = t^4 - 4\ln t, \quad y = 4t^2.$$

Show that the length of the arc of C from the point where $t = 2$ to the point where $t = 4$ is

$$240 + 4\ln 2.$$ [5]

2 Let $y = e^x$. Find the mean value of y with respect to x over the interval $0 \leqslant x \leqslant 2$. [2]

Show that the mean value of x with respect to y over the interval $1 \leqslant y \leqslant e^2$ is $\dfrac{e^2 + 1}{e^2 - 1}$. [4]

3 The curve C has polar equation

$$r = \left(\tfrac{1}{2}\pi - \theta\right)^2,$$

where $0 \leqslant \theta \leqslant \frac{1}{2}\pi$. Draw a sketch of C. [3]

Find the area of the region bounded by C and the initial line, leaving your answer in terms of π. [3]

4 The matrix $\mathbf{A}$ has λ as an eigenvalue with $\mathbf{e}$ as a corresponding eigenvector. Show that $\mathbf{e}$ is an eigenvector of $\mathbf{A}^2$ and state the corresponding eigenvalue. [3]

Given that one eigenvalue of $\mathbf{A}$ is 3, find an eigenvalue of the matrix $\mathbf{A}^4 + 3\mathbf{A}^2 + 2\mathbf{I}$, justifying your answer. [3]

5 The curve C has equation

$$x^2 - xy - 2y^2 = 4.$$

Show that, at the point $A\,(2, 0)$ on C, $\dfrac{dy}{dx} = 2$. [2]

Find the value of $\dfrac{d^2y}{dx^2}$ at A. [5]

6 The matrix $\mathbf{A}$ is defined by

$$\mathbf{A} = \begin{pmatrix} 1 & -1 & -2 & -3 \\ -2 & 1 & 7 & 2 \\ -3 & 3 & 6 & \alpha \\ 7 & -6 & -17 & -17 \end{pmatrix}.$$

(i) Show that if $\alpha = 9$ then the rank of $\mathbf{A}$ is 2, and find a basis for the null space of $\mathbf{A}$ in this case. [5]

(ii) Find the rank of $\mathbf{A}$ when $\alpha \neq 9$. [2]

7 Let $I_n = \int_0^1 \frac{1}{(1+x^4)^n}\,\mathrm{d}x$. By considering $\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{x}{(1+x^4)^n}\right)$, show that

$$4nI_{n+1} = \frac{1}{2^n} + (4n-1)I_n.$$ [4]

Given that $I_1 = 0.866\,97$, correct to 5 decimal places, find I_3. [4]

8 Find y in terms of t, given that

$$5\frac{\mathrm{d}^2y}{\mathrm{d}t^2} + 6\frac{\mathrm{d}y}{\mathrm{d}t} + 5y = 15 + 12t + 5t^2,$$

and that $y = \frac{\mathrm{d}y}{\mathrm{d}t} = 0$ when $t = 0$. [9]

9 Use induction to prove that

$$\sum_{n=1}^{N} \frac{4n+1}{n(n+1)(2n-1)(2n+1)} = 1 - \frac{1}{(N+1)(2N+1)}.$$ [6]

Show that

$$\sum_{n=N+1}^{2N} \frac{4n+1}{n(n+1)(2n-1)(2n+1)} < \frac{3}{8N^2}.$$ [4]

10 Use de Moivre's theorem to express $\cos 8\theta$ as a polynomial in $\cos\theta$. [5]

Hence

(i) express $\cos 8\theta$ as a polynomial in $\sin\theta$, [2]

(ii) find the exact value of

$$4x^4 - 8x^3 + 5x^2 - x,$$

where $x = \cos^2\left(\frac{1}{8}\pi\right)$. [3]

11 The plane Π_1 has equation

$$\mathbf{r} = \mathbf{i} + 2\mathbf{j} + \mathbf{k} + \theta(2\mathbf{j} - \mathbf{k}) + \phi(3\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}).$$

Find a vector normal to Π_1 and hence show that the equation of Π_1 can be written as $2x + 3y + 6z = 14$. [4]

The line l has equation

$$\mathbf{r} = 3\mathbf{i} + 8\mathbf{j} + 2\mathbf{k} + t(4\mathbf{i} + 6\mathbf{j} + 5\mathbf{k}).$$

The point on l where $t = \lambda$ is denoted by P. Find the set of values of λ for which the perpendicular distance of P from Π_1 is not greater than 4. [4]

The plane Π_2 contains l and the point with position vector $\mathbf{i} + 2\mathbf{j} + \mathbf{k}$. Find the acute angle between Π_1 and Π_2. [4]

12 Answer only **one** of the following two alternatives.

EITHER

The curve C has equation

$$y = \frac{(x-2)(x-a)}{(x-1)(x-3)},$$

where a is a constant not equal to 1, 2 or 3.

(i) Write down the equations of the asymptotes of C. [2]

(ii) Show that C meets the asymptote parallel to the x-axis at the point where $x = \frac{2a-3}{a-2}$. [2]

(iii) Show that the x-coordinates of any stationary points on C satisfy

$$(a-2)x^2 + (6-4a)x + (5a-6) = 0,$$

and hence find the set of values of a for which C has stationary points. [6]

(iv) Sketch the graph of C for

(a) $a > 3$,

(b) $2 < a < 3$.

[4]

OR

The roots of the equation

$$x^4 - 5x^2 + 2x - 1 = 0$$

are $\alpha, \beta, \gamma, \delta$. Let $S_n = \alpha^n + \beta^n + \gamma^n + \delta^n$.

(i) Show that

$$S_{n+4} - 5S_{n+2} + 2S_{n+1} - S_n = 0.$$ [2]

(ii) Find the values of S_2 and S_4. [3]

(iii) Find the value of S_3 and hence find the value of S_6. [6]

(iv) Hence find the value of

$$\alpha^2(\beta^4 + \gamma^4 + \delta^4) + \beta^2(\gamma^4 + \delta^4 + \alpha^4) + \gamma^2(\delta^4 + \alpha^4 + \beta^4) + \delta^2(\alpha^4 + \beta^4 + \gamma^4).$$ [3]