



UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS  
General Certificate of Education  
Advanced Subsidiary Level and Advanced Level

**MATHEMATICS**

**9709/12**

Paper 1 Pure Mathematics 1 (P1)

**October/November 2009**

**1 hour 45 minutes**

Additional Materials:      Answer Booklet/Paper  
   Graph Paper  
   List of Formulae (MF9)



**READ THESE INSTRUCTIONS FIRST**

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.

Write your Centre number, candidate number and name on all the work you hand in.

Write in dark blue or black pen.

You may use a soft pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 75.

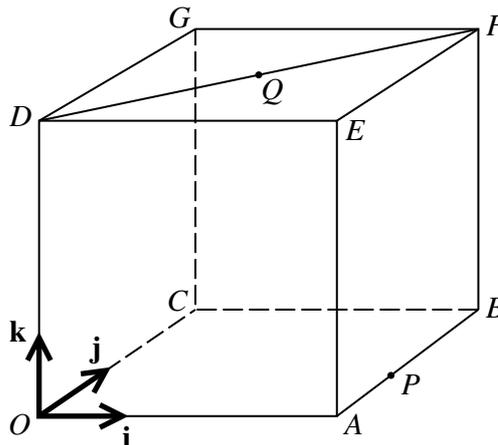
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

This document consists of **4** printed pages.



- 1 The equation of a curve is such that  $\frac{dy}{dx} = \frac{3}{\sqrt{x}} - x$ . Given that the curve passes through the point (4, 6), find the equation of the curve. [4]
- 2 (i) Find, in terms of the non-zero constant  $k$ , the first 4 terms in the expansion of  $(k + x)^8$  in ascending powers of  $x$ . [3]
- (ii) Given that the coefficients of  $x^2$  and  $x^3$  in this expansion are equal, find the value of  $k$ . [2]
- 3 A progression has a second term of 96 and a fourth term of 54. Find the first term of the progression in each of the following cases:
- (i) the progression is arithmetic, [3]
- (ii) the progression is geometric with a positive common ratio. [3]
- 4 The function  $f$  is defined by  $f : x \mapsto 5 - 3 \sin 2x$  for  $0 \leq x \leq \pi$ .
- (i) Find the range of  $f$ . [2]
- (ii) Sketch the graph of  $y = f(x)$ . [3]
- (iii) State, with a reason, whether  $f$  has an inverse. [1]
- 5 (i) Prove the identity  $(\sin x + \cos x)(1 - \sin x \cos x) \equiv \sin^3 x + \cos^3 x$ . [3]
- (ii) Solve the equation  $(\sin x + \cos x)(1 - \sin x \cos x) = 9 \sin^3 x$  for  $0^\circ \leq x \leq 360^\circ$ . [3]

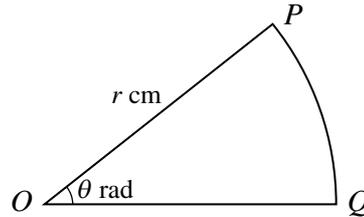
6



In the diagram,  $OABCDEFG$  is a cube in which each side has length 6. Unit vectors  $\mathbf{i}$ ,  $\mathbf{j}$  and  $\mathbf{k}$  are parallel to  $\overrightarrow{OA}$ ,  $\overrightarrow{OC}$  and  $\overrightarrow{OD}$  respectively. The point  $P$  is such that  $\overrightarrow{AP} = \frac{1}{3}\overrightarrow{AB}$  and the point  $Q$  is the mid-point of  $DF$ .

- (i) Express each of the vectors  $\overrightarrow{OQ}$  and  $\overrightarrow{PQ}$  in terms of  $\mathbf{i}$ ,  $\mathbf{j}$  and  $\mathbf{k}$ . [3]
- (ii) Find the angle  $OQP$ . [4]

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A piece of wire of length 50 cm is bent to form the perimeter of a sector  $POQ$  of a circle. The radius of the circle is  $r$  cm and the angle  $POQ$  is  $\theta$  radians (see diagram).

- (i) Express  $\theta$  in terms of  $r$  and show that the area,  $A$  cm<sup>2</sup>, of the sector is given by

$$A = 25r - r^2. \quad [4]$$

- (ii) Given that  $r$  can vary, find the stationary value of  $A$  and determine its nature. [4]

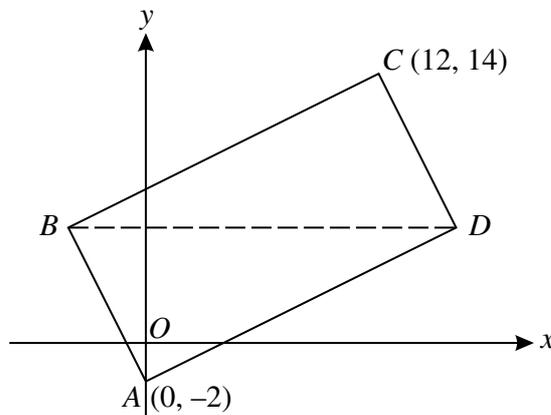
- 8 The function  $f$  is such that  $f(x) = \frac{3}{2x+5}$  for  $x \in \mathbb{R}$ ,  $x \neq -2.5$ .

- (i) Obtain an expression for  $f'(x)$  and explain why  $f$  is a decreasing function. [3]

- (ii) Obtain an expression for  $f^{-1}(x)$ . [2]

- (iii) A curve has the equation  $y = f(x)$ . Find the volume obtained when the region bounded by the curve, the coordinate axes and the line  $x = 2$  is rotated through  $360^\circ$  about the  $x$ -axis. [4]

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The diagram shows a rectangle  $ABCD$ . The point  $A$  is  $(0, -2)$  and  $C$  is  $(12, 14)$ . The diagonal  $BD$  is parallel to the  $x$ -axis.

- (i) Explain why the  $y$ -coordinate of  $D$  is 6. [1]

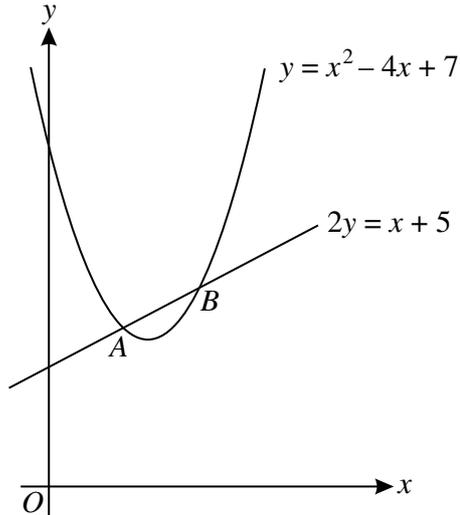
The  $x$ -coordinate of  $D$  is  $h$ .

- (ii) Express the gradients of  $AD$  and  $CD$  in terms of  $h$ . [3]

- (iii) Calculate the  $x$ -coordinates of  $D$  and  $B$ . [4]

- (iv) Calculate the area of the rectangle  $ABCD$ . [3]

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- (i) The diagram shows the line  $2y = x + 5$  and the curve  $y = x^2 - 4x + 7$ , which intersect at the points  $A$  and  $B$ . Find
- (a) the  $x$ -coordinates of  $A$  and  $B$ , [3]
- (b) the equation of the tangent to the curve at  $B$ , [3]
- (c) the acute angle, in degrees correct to 1 decimal place, between this tangent and the line  $2y = x + 5$ . [3]
- (ii) Determine the set of values of  $k$  for which the line  $2y = x + k$  does not intersect the curve  $y = x^2 - 4x + 7$ . [4]

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