CAMBRIDGE INTERNATIONAL EXAMINATIONS

Cambridge International Advanced Level

MARK SCHEME for the May/June 2015 series

9280 MATHEMATICS (US)

9280/31 Paper 3 (Paper 3), maximum raw mark 75

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the May/June 2015 series for most Cambridge IGCSE®, Cambridge International A and AS Level components and some Cambridge O Level components.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1	Use law for the logarithm of a power at least once	*M1	
	Obtain correct linear equation, e.g. $5x\ln 2 = (2x+1)\ln 3$	A1	
	Solve a linear equation for x	M1 dep *M...	
	Obtain $x = 0.866$	A1	[...
2	Attempt calculation of at least 3 ordinates	M1	
	Obtain 9, 7, 1, 17	A1	
	Use trapezium rule with $h = 1$	M1	
	Obtain $\frac{1}{2}(9+14+2+17)$ or equivalent and hence 21	A1	[4]
3	Either Obtain correct (unsimplified) version of x^2 or x^4 term in $(1-2x^2)^{-2}$	M1	
	Obtain $1+4x^2$	A1	
	Obtain … $+12x^4$	A1	
	Obtain correct (unsimplified) version of x^2 or x^4 term in $(1+6x^2)^{\frac{2}{3}}$	M1	
	Obtain $1+4x^2-4x^4$	A1	
	Combine expansions to obtain $k = 16$ with no error seen	A1	
	Or Obtain correct (unsimplified) version of x^2 or x^4 term in $(1+6x^2)^{\frac{2}{3}}$	M1	
	Obtain $1+4x^2$	A1	
	Obtain … $-4x^4$	A1	
	Obtain correct (unsimplified) version of x^2 or x^4 term in $(1-2x^2)^{-2}$	M1	
	Obtain $1+4x^2+12x^4$	A1	
	Combine expansions to obtain $k = 16$ with no error seen	A1	[6]
4	Differentiate to obtain form $a\sin 2x + b\cos x$	M1	
	Obtain correct $-6\sin 2x + 7\cos x$	A1	
	Use identity $\sin 2x = 2\sin x\cos x$	B1	
	Solve equation of form $c\sin x\cos x + d\cos x = 0$ to find at least one value of x	M1	
	Obtain 0.623	A1	
	Obtain 2.52	A1	
	Obtain 1.57 or $\frac{1}{2}\pi$ from equation of form $c\sin x\cos x + d\cos x = 0$	A1	
	Treat answers in degrees as MR – 1 situation		[7]

5	(a)	Use identity $\tan^2 2x = \sec^2 2x - 1$	B1	
		Obtain integral of form $ax + b\tan 2x$	M1	
		Obtain correct $3x + \frac{1}{2}\tan 2x$, condoning absence of + c	A1	[
	(b)	State $\sin x \cos\frac{1}{2}\pi + \cos x \ \sin\frac{1}{6}\pi$	B1	
		Simplify integrand to $\cos\frac{1}{6}\pi + \frac{\cos x \sin\frac{1}{6}\pi}{\sin x}$ or equivalent	B1	
		Integrate to obtain at least term of form $a\,\mathrm{In}(\sin x)$	*M1	
		Apply limits and simplify to obtain two terms	M1 dep *M	
		Obtain $\frac{1}{8}\pi\sqrt{3} - \frac{1}{2}\ln(\frac{1}{\sqrt{2}})$ or equivalent	A1	[5]
6	(i)	Obtain $\pm\begin{pmatrix}2\\-3\\-4\end{pmatrix}$ as direction vector of l_1	B1	
		State that two direction vectors are not parallel	B1	
		Express general point of l_1 or l_2 in component form, e.g. $(2\lambda, 1-3\lambda, 5-4\lambda)$ or $(7+\mu, l+2\mu, 1+5\mu)$	B1	
		Equate at least two pairs of components and solve for λ or for μ	M1	
		Obtain correct answers for λ and μ	A1	
		Verify that all three component equations are not satisfied (with no errors seen)	A1	[6]
	(ii)	Carry out correct process for evaluating scalar product of $\begin{pmatrix}1\\2\\5\end{pmatrix}$ and $\begin{pmatrix}1\\0\\0\end{pmatrix}$	M1	
		Use correct process for finding modulus and evaluating inverse cosine	M1	
		Obtain $79.5°$ or 1.39 radians	A1	[3]
7		Separate variables and factorise to obtain $\frac{\mathrm{d}y}{(3y+1)(y+3)} = 4x\,\mathrm{d}x$ or equivalent	B1	
		State or imply the form $\frac{A}{3y+1} + \frac{B}{y+3}$ and use a relevant method to find A or B	M1	
		Obtain $A = \frac{3}{8}$ and $B = -\frac{1}{8}$	A1	
		Integrate to obtain form $k_1\ln(3y+1) + k_2\ln(y+3)$	M1	
		Obtain correct $\frac{1}{8}\ln(3y+1) - \frac{1}{8}\ln(y=3) = 2x^2$ or equivalent	A1	
		Substitute $x = 0$ and $y = 1$ in equation of form $k_1\ln(3y+1) + k_2\ln(y+3) = k_3x^2 + c$ to find a value of c	M1	
		Obtain $c = 0$	A1	
		Use correct process to obtain equation without natural logarithm present	M1	
		Obtain $y = \frac{3e^{16x^2} - 1}{3 - e^{16x^2}}$ or equivalent	A1	[9]

8	(i)	Either	Expand $(2-\mathrm{i})^2$ to obtain $3-4\mathrm{i}$ or unsimplified equivalent	B1	
			Multiply by $\frac{3+4\mathrm{i}}{3+4\mathrm{i}}$ and simplify to $x+\mathrm{i}y$ form or equivalent	M1	
			Confirm given answer $2+4\mathrm{i}$	A1	
		Or	Expand $(2-\mathrm{i})^2$ to obtain $3-4\mathrm{i}$ or unsimplified equivalent	B1	
			Obtain two equations in x and y and solve for x or y	M1	
			Confirm given answer $2+4\mathrm{i}$	A1	[3]
	(ii)		Identify $4+4$ or $-4+4\mathrm{i}$ as point at either end or state $p=2$ or state $p=-6$	B1	
			Use appropriate method to find both critical values of p	M1	
			State $-6 \leqslant p \leqslant 2$	A1	[3]
	(iii)		Identify equation as of form $\lvert z-a \rvert = a$ or equivalent	M1	
			Form correct equation for a not involving modulus, e.g. $(a-2)^2+4^2=a^2$	A1	
			State $\lvert z-5 \rvert = 5$	A1	[3]
9	(i)		Use product rule to find first derivative	M1	
			Obtain $2x\mathrm{e}^{2-x} - x^2\mathrm{e}^{2-x}$	A1	
			Confirm $x=2$ at M	A1	[3]
	(ii)		Attempt integration by parts and reach $\pm x^2\mathrm{e}^{2-x} \pm \int 2x\mathrm{e}^{2-x}\,\mathrm{d}x$	*M1	
			Obtain $-x^2\mathrm{e}^{2-x} + \int 2x\mathrm{e}^{2-x}\,\mathrm{d}x$	A1	
			Attempt integration by parts and reach $\pm x^2\mathrm{e}^{2-x} \pm 2x\mathrm{e}^{2-x} \pm 2\mathrm{e}^{2-x}$	*M1	
			Obtain $-x^2\mathrm{e}^{2-x} - 2x\mathrm{e}^{2-x} - 2\mathrm{e}^{2-x}$	A1	
			Use limits 0 and 2 having integrated twice	M1 dep *M	
			Obtain $2\mathrm{e}^2 - 10$	A1	[6]
10	(i)		Obtain $\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{2}{t+2}$ and $\frac{\mathrm{d}y}{\mathrm{d}t} = 3t^2+2$	B1	
			Use $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}t} \div \frac{\mathrm{d}x}{\mathrm{d}t}$	M1	
			Obtain $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{2}(3t^2+2)(t+2)$	A1	
			Identify value of t at the origin as –1	B1	
			Substitute to obtain $\frac{5}{2}$ as gradient at the origin	A1	[5]
	(ii)	(a)	Equate derivative to $\frac{1}{2}$ and confirm $p = \frac{1}{3p^2+2} - 2$	B1	[1]
		(b)	Use the iterative formula correctly at least once	M1	
			Obtain value $p=-1.924$ or better (–1.92367…)	A1	
			Show sufficient iterations to justify accuracy or show a sign change in appropriate interval	A1	
			Obtain coordinates (–5.15, –7.97)	A1	[4]