Cambridge International Examinations
Cambridge International Advanced Level

THINKING SKILLS **9694/31**

Paper 3 Problem Analysis and Solution **May/June 2016**

MARK SCHEME

Maximum Mark: 50

Published

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1 (a) What is the lowest price that Peter could charge for a Medium Luxury pizza? [1]

$3.96

Total cost to Peter will be $1.80 to make the pizza, plus an additional $1.50 to cover the estimated delivery charge. Total is $3.30. With 20% as profit he would need to charge an extra $0.66.

(b) If the prices are still set as before, what is the greatest percentage discount (to the nearest 1%) that Peter can offer on orders of 3 pizzas? [3]

22%

It is the fact that the delivery charge is only paid once that means that the discount can be offered, so the discount must not amount to more than $3.60 ($3 + 20% that has been charged within the price, but is not a cost incurred by Peter). The discount possible will be limited by the amount that can be offered on the most expensive pizzas. The most expensive pizza will be sold for $5.40, so the discount is made on a total cost of $16.20. The discount is therefore 22(.2…)%.

Alternatively:
The amount of the discount must be determined by the amount that can be offered on the largest pizzas:
3 Large Luxury pizzas will cost 3 × $3.00 + $1.50 = $10.50 to make and deliver, so a total charge of $12.60 is needed.
Without a discount the total charge would be 3 × $5.40 = $16.20, so the discount can be up to $3.60, which is 22%.

If three marks cannot be awarded, award 1 mark for each of the following (max 2):

- *Identifying that the size of the discount is determined by the price of the most expensive pizza.*
- *Calculating the total cost that Peter pays to make and deliver three pizzas of the same size.*
- *Calculating the maximum discount for any individual order.*

(c) What price should Peter now charge for each Large Basic pizza so that he will still make a 20% profit on a delivery of 2 such pizzas if the second one is half price? [2]

Total cost to Peter will be $3.20 to make the pizzas and $1.50 for the delivery, making a total of $4.70. To achieve a 20% profit a total charge of $5.64 [1 mark] is required.
This would be achieved by having the pizza priced at $3.76 (with the second pizza at $1.88).

1 mark for clear working which shows their total charge (cost + delivery + profit) ÷ 1.5 to give the final price.
OR
1 mark for correct calculation for another pizza (must be stated).
OR
1 mark for correct phrasing of the problem algebraically:
$p + 0.5p = 1.2(c + c + 1.5)$
where p is the price, and c is the cost of making.

(d) If a customer placed an order for two Large Standard pizzas and one other, cheaper, pizza, what price would the other pizza have to be so that the two offers resulted in the same overall cost for the order? [2]

Two Large Standard pizzas would be priced at \$10.40. Therefore the two different charges for the order would be:

- \$10.40 + half the price of the other pizza (one pizza half price)
- \$9.36 + 90% of the price of the other pizza (10% discount applied)

Therefore 40% of the price of the other pizza must be \$1.04.
The other pizza would need to be priced at \$2.60.

1 mark for identifying that 90% of the price of two Large Standard pizzas is \$9.36
OR
1 mark for an algebraic parsing of the problem:
$0.9(p + p + x) = p + p + 0.5x$

SC1: If working on basis that the other pizza is priced higher than the Large Standard, answer of \$15.60.

Trial and improvement solutions – award 1 mark for a correct comparison of two discounted prices and an adjustment in the right direction.

(e) If Peter does apply both offers to this order, what will be his overall profit? [2]

\$2.70 *(accept 30%)*

The total cost to Peter would be \$7.50 for the pizzas, plus the \$1.50 for the delivery, so a total of \$9.
The price of the order before the discount would be \$10.40 for two pizzas at full price plus \$2.60 for one pizza at half price.
The total is \$13 and the 10% discount brings this down to \$11.70 (a profit of \$2.70 or 30%).

If 2 marks cannot be awarded, award 1 mark for correctly calculating that the order will cost the customer a total of \$11.70, or clear, correct working with one arithmetic error.

2 (a) (i) The first car to exit came from car park C and the second from car park D. From which car parks did the next seven cars come from? List them in order. [2]

(C D) A E B D A E C

1 mark for D and Es in the correct positions OR As, B and C in the correct positions
OR
1 mark for the first four (AEBD) correct

(ii) Which car parks empty long before the others? [1]

A, D, E

(b) Explain why this will not be possible. [1]

There is only one exit.

(c) Determine which junction(s) should be 2 to 1, and which branch is which, to achieve Terry's aim. Explain your reasoning for junction Z. [3]

Junction Y and junction Z only should be 2–1 [1 mark].

At junction Y: 2 cars from W for every 1 from A [1 mark]

Justification [1 mark]:

At junction Z:

1–1 would have 6 from D leaving in same time as 4 from A
2–1 would have 3 from D leaving in same time as 4 from A
(1–2 would have 6 from D in the same time as 2 from A)
3/7 is closer to 1/2 than 2/5, so best balance is 2 from Y : 1 from X

OR

1–1 would mean a ratio of 3 : 2, 1–2 a ratio of 1 : 3, and 2–1 a ratio of 3 : 4.
3 : 4 is the narrowest ratio, so 2–1 (Y : X) is the best arrangement.

OR

1–1 means that D, E would empty first, after 48 minutes
2–1 means that A, B, C would empty first, after 54 minutes

OR

Listing car parks of origin of at least 12 cars for the 1 : 1 case, and 18 cars for the 2 : 1 case.

(d) (i) Draw a different layout for the tracks, with no more than two tracks merging at any junction, which is not equivalent to re-labelling some of the car parks. [1]

Any scheme which does not have 3 meeting 2 is necessary and sufficient to be different. So accept any adjustment of either scheme where 4 meet 1.

It would be of the form ((AB) (CD)) E or (((AB) C) D) E.

(ii) How long would it take before one of the car parks becomes empty in your new layout, if cars merge alternately at all junctions? [2]

The singleton will empty in twice the time it would if there were no other cars. Since it takes an hour to empty all five, one would take 12 minutes and so here it must be 24 minutes. (The rest will only empty on the hour.)

If 2 marks cannot be awarded, 1 mark for working noting 12 minutes

Award 2 marks for correct follow-through (FT) from ***(d)(i)****, provided there is more than one junction*

3 (a) (i) If all 20 volunteers were occupied with the fun run, what would be the maximum number of runners that could take part? [2]

(20 – 1 – 6) × 50 = <u>650</u>

Award 1 mark for (20 –1– 6), or the director is forgotten (yielding 700 contestants).

(ii) If this maximum number of runners all wanted a cup of tea in the café immediately after the race, how many more volunteers would be needed? [1]

650/20 = 32.5; 33 volunteers needed; 33 – 20 = <u>13</u> more volunteers

Follow-through (FT) from any number in ***(a)(i)****.*

(b) What is the maximum number of runners that can take part in the race? [3]

With 1 director and 6 marshals involved in the race, there are 13 volunteers available.
Every 50 runners requires 1 first-aider; every 20 visitors to the café requires 1 server.
100 runners require 2 + 5 volunteers.
200 runners require 4 + 10 volunteers.
180 runners require 4 + 9 = 13 volunteers, so <u>180</u> is the maximum.

2 marks for a solution between 150 and 200 (inclusive) with details of feasible staffing
1 mark for a solution between 50 and 600 (inclusive), with details of feasible staffing

OR

Algebraic 'critical values' approach:
(c/50 + 7) + c/20 = 20 [1 mark] yielding 185.7… [1 mark]
OR
20x = 50(13–x) [1 mark] yielding 9.2857… [1 mark]

(c) Will the addition of the volunteers involved in the race be sufficient to serve all the visitors in the café after the race? [2]

The 180 runners will require 9 servers. *FT from* ***(b)***
There are 11 more volunteers now available after the race.
So Yes.

1 mark for 9 volunteers and 11 volunteers;
1 mark (dependent upon working) for conclusion 'yes'
OR
1 mark for 400 (possible) and 360 (actual);
1 mark (dependent upon working) for conclusion 'yes'

Answer of '18, so Yes' (derived from 360/20 < 20) scores 2 marks.

(d) (i) What is the minimum total number of hours of volunteering time that could be needed for the event, assuming that at least one runner takes part in the fun run? [1]

5 hours (café) + (8 × 2 =) 16 hours (race) = 21 hours

(ii) What is the total number of hours of volunteering time that will be needed for the event, assuming that the organisers allow as many runners to come as possible? [3]

180 runners:
Before the race: 1 hour of 1 café staff = 1
AND During the race: 2 hours of (9 café staff + 11 race staff) = 40
After the race: 2 hours of 18 café staff = 36
77 hours of volunteering time.

1 mark for 22 hours needed for the race (FT)
1 mark for two of the three components of the café-staffing:
(1 × 1 = 1; 2 × 9 = 18; 2 × 18 = 36 OR 1 × 5 = 5; 2 × 8 = 16; 2 × 17 = 34)
1 mark for 77 hours

(e) What is the smallest number of equal slices that each pie should be divided into so that the runners and supporters could each get one slice? [3]

16 slices

100 – 77 = 23 hours of time baking [1 mark] *(FT from **(d)(ii)**)*
360 ÷ 23 = 15.65 slices *(FT from **(b)**)*
16 slices [1 mark] *(correct answer only)*

4 (a) Name the three judges who awarded sets of points at the rehearsal that would not have been allowed at the contest itself, and give the number of the instruction that each one has violated. [3]

Jim – instruction number 1 (31 points awarded)
Jane – instruction number 5 (difference between second-placed and third-placed bands only 1 point)
Joe – instruction number 3 (two bands awarded the same number of points)

Award 1 mark for each correct instruction number linked to the relevant judge.

SC1: *three judges correctly identified with no reference to instructions.*

(b) What is the greatest number of points that a judge could award to one band? [1]

26 (the set of points would be 26, 3, 1)

(c) (i) Explain why a judge could not award points to all seven bands. [2]

A minimum of 32 points would be required (1+2+3+4+5+7+10) if all the other instructions are fulfilled.
OR
1,2,3,4,5,7,x is the minimum to conform to rules 2, 3 and 5. x would have to score 8 to conform to rule 1. This breaks rule 4.

Award 1 mark for a single defaulting case without comment about generalizing.

SC1: 32 points would be required

(ii) List all the sets of points that a judge could award to six of the bands. [3]

14, 6, 4, 3, 2, 1
13, 7, 4, 3, 2, 1
12, 8, 4, 3, 2, 1
12, 7, 5, 3, 2, 1
11, 8, 5, 3, 2, 1
11, 7, 5, 4, 2, 1
10, 7, 5, 4, 3, 1

Award 3 marks for all 7 sets, without any incorrect sets.
If 3 marks cannot be awarded, award 2 marks for at least 5 correct sets, and at most 2 incorrect sets
If 2 marks cannot be awarded, award 1 mark for at least 2 correct sets, and ignore incorrect sets.

(d) According to the incorrect scoreboard, which band appeared to have won the contest? [1]

Captain Cook

(e) (i) Which three bands did Michael award points to? [1]

The Elbees, Ashwin and The Urn (otherwise 15 and/or 17 and/or 18 would have appeared twice on the scoreboard).

(ii) Deduce the final totals of all seven bands. [4]

The Elbees	23
DRS	0
Ashwin	31
The Urn	26
Riverside	22
Captain Cook	28
Broadsword	20

If 4 marks cannot be awarded, award 1 mark for evidence of appreciation of each of the following (max 3):

- The set of points must be 14, 11 and 5.
- 14 points were awarded to Ashwin (otherwise 28 or 22 would have appeared twice on the scoreboard).
- (So) 11 points were awarded to The Urn (otherwise 20 would have appeared twice on the scoreboard).
- 5 points were awarded to the Elbees (only possible allocation of score less than 10)