THINKING SKILLS   **9694/33**

Paper 3 Problem Analysis and Solution   **May/June 2016**

                                                                                      2 hours

Additional Materials: Electronic Calculator

READ THESE INSTRUCTIONS FIRST

An answer booklet is provided inside this question paper. You should follow the instructions on the front cover of the answer booklet. If you need additional answer paper ask the invigilator for a continuation booklet.

Answer **all** the questions.

Show your working. Marks may be awarded for correct steps towards a solution, even if the final answer is not correct. Marks may be lost if working needed to support an answer is not shown.

Calculators should be used where appropriate.

The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **7** printed pages, **1** blank page and **1** insert.

International Examinations

[Turn over

1   Bill owns three stores which sell models. Every Monday, Bill sends supplies to the stores from his warehouse. All of the supplies are packaged into boxes of the same size and 20 of these boxes can be loaded onto the van for each delivery.

The distances, in kilometres, between Bill's warehouse and the three stores are shown below.

Bill's Warehouse			
5	Model Solutions		
7	8	Model Emporium	
8	6	10	Model Market

Bill sometimes needs to make more than one trip when delivering the boxes. He wants to know the total distance that he needs to travel for any of the trips that deliver to just two of the stores before returning to the warehouse. The three trips are:

Warehouse → Model Solutions → Model Emporium → Warehouse
Warehouse → Model Solutions → Model Market → Warehouse
Warehouse → Model Emporium → Model Market → Warehouse

Bill has already worked out that he needs to travel a total distance of 20 km if he makes the first trip.

(a) What are the total distances that Bill needs to travel to make each of the other trips?   [2]

This morning, Bill needs to deliver 12 boxes to Model Solutions, 8 boxes to Model Emporium and 9 boxes to Model Market. He cannot load all of the boxes onto the van, so Bill has decided to deliver to one of the stores first and then return to the warehouse where he will load all the boxes for the other two stores. When he has completed the deliveries he will return the van to the warehouse.

(b) Which store should Bill deliver to first in order to make the total distance travelled as short as possible? What is this shortest distance?   [3]

Bill hires the van at a cost of $6 per km travelled. It is also possible for him to hire a larger van, which would hold 25 boxes, but this would cost $7 per km travelled.

Bill already knows that he will need to deliver 13 boxes to Model Solutions and 12 boxes to Model Market next week, but does not yet know how many boxes will need to be delivered to Model Emporium.

(c) (i) If Model Emporium does **not** require any boxes delivered next week, how much would Bill save by hiring the larger van rather than the smaller one?   [2]

(ii) If Model Emporium does require some boxes delivered next week, what is the smallest number of boxes that could be needed at Model Emporium to make it cheaper for Bill to hire the larger van?   [3]

2 Gwen runs a company called *Lance-a-lock* and wishes to reassure her customers about the security of the locks she sells. The most basic type of lock is defined by a 5-digit code. The digits of the code represent the shape of the key that matches the lock by describing it as the lengths of 5 columns of squares, each of which can be up to 4 units long. An example is shown below.

1 4 0 2 2
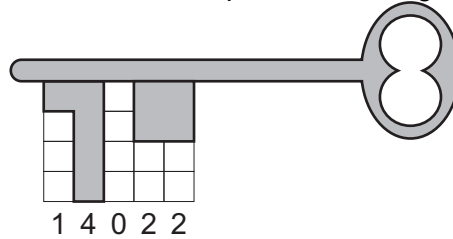
The columns of the key can be any integer length from 0 to 4 – except that the extreme case of **0 0 0 0 0** is not permissible because it would not turn any lock.

(a) How many different keys of this sort can be made? [2]

The Guild of Locksmiths places restrictions on what locks and keys are permitted. One is that no lock should have a 'low security code' – one which could be opened by a key with a code that does not match the lock, by partially inserting the key.

An example of a lock with a low security code is **0 2 0 0 0**. This could be opened by the key with code **2 0 0 0 0**, partially inserted so that the **2** lines up with the appropriate part of the lock.

You should assume that the key cannot be inserted further into the lock than intended – so the key with code **0 2 0 0 0** cannot open the lock with code **2 0 0 0 0**.

(b) Which five locks could be opened by the key **3 0 0 0 0**? [1]

(c) Give an example of a key, with a code containing no more than two **0**s, that could open more than one lock. List all the locks that this key could open. [2]

The Guild of Locksmiths has a rule which prevents low security locks being made, but does not exclude any others. This rule places a constraint on **one** of the digits in the code for the lock.

(d) (i) What is this constraint? [1]

(ii) How many locks are permissible, given this rule? [1]

For some locks (such as door locks) the key can be inserted from either side. In these cases the shape of the key needs to match the shape of the lock whichever side it is put in.

(e) How many locks, including low security locks, are permissible given this restriction? [1]

Considering also the restriction on low security locks, Gwen concludes that there are only 100 locks available using this kind of key, and thinks that this may worry some of her customers. She therefore considers using locks with 7-digit codes.

(f) Subject to all the restrictions, how many locks with 7-digit codes are permissible? [2]

3 An archaeologist has found some coins from a Bronze Age village, and is trying to establish what values they could have had. There are three sizes of coin: Small (S), Medium (M) and Large (L). He has also found some records of the prices of goods sold, given as numbers of these coins.

He makes the following assumptions:

- The larger the coin, the more it is worth.
- The three coin sizes he has found were the smallest three types used.
- When writing a price, the bigger coins are always used where possible. So an item with a price of 5 Smalls (for example) would be evidence that a Medium could not be worth 5 Smalls or less.
- No price ever requires more than 9 of any type of coin.
- The Small coin had a value of 1.

(a) If the Medium coin had a value of 5 and the Large coin had a value of 19, how would the price of an item with a value of 30 be written, in terms of L, M and S? [1]

(b) If a price of 2L, 0M, 1S can be paid for precisely with 1 Large, 3 Medium and 4 Small coins, give one example of possible values for the Medium and Large coins. [2]

(c) The first price found was for a jug priced 3M, 6S. What are the only four possible values for this jug? [2]

(d) Some time later, a necklace priced 1L, 6M, 7S was found.

  (i) What is the largest possible value for this necklace? [1]

  (ii) What is the smallest possible value for this necklace? [2]

  (iii) Initially, the archaeologist believes the necklace has a value of 118. Deduce what the values of the Medium and Large coins would be if this were the case. [2]

  (iv) What is the smallest possible value for the necklace which would **not** allow the values of the Medium and Large coins to be deduced? [3]

  (v) What value for the necklace could still be explained by three different values of the Large coin? [2]

[Question 4 begins on the next page]

 [Turn over

4   Thursday is Quiz Night at Playard Sports Club, with teams competing in a league.

Each league season is 13 weeks, at the end of which the top team wins $250. There is also a $50 prize for each week's highest final score.

To help spread the prize money around, and give the weaker teams some encouragement, a handicap system is in operation. Every team that wins the $50 weekly prize is also given a handicap of 2 points. This means that 2 points must be deducted from the team's question points every week for the rest of the season. Teams who win more than once will have their handicap increased each time. All teams start a season with no handicap.

If two or more teams tie for first place any week, a tie-break is required. No further points are scored, but the $50 prize is awarded to the team that wins the tie-break. All teams that lose a tie-break receive a handicap of 1 point for the rest of the season.

Each quiz consists of 60 general knowledge questions, worth 1 point each, and 20 picture questions, worth 2 points each.

In order not to discourage teams that miss an occasional week, only a team's 10 best final scores during a season count towards the overall league total.

Last night was week 12 of the season. All ten teams were present, and the results were as follows:

Team	Question points	Handicap	Final score
McGann	84	2	82
Troughton	85	4	81
Tennant	82	3	79
Baker	82	4	78
Eccleston	83	5	78
Pertwee	77	0	77
Hartnell	79	4	75
McCoy	74	2	72
Davison	72	0	72
Smith	70	0	70

The **final** scores of the teams in the previous weeks of this season and the league totals after week 11 are shown below.

Team	Week													League Total
	1	2	3	4	5	6	7	8	9	10	11	12	13	
Eccleston	74	71	76	85	79	83	78	84	73	76	82			790
McCoy	80	81	84	82	76	77	80	74	73	79	74			787
Tennant	75	71	78	74	79	83	84	77	70	72	75			768
Baker	77	75	69	73	–	83	81	74	68	82	77			759
Davison	72	68	75	72	70	–	67	73	71	78	78			724
Troughton	83	–	80	72	76	–	79	81	74	80	78			703
McGann	77	83	74	76	81	80	73	–	–	75	82			701
Hartnell	69	84	75	–	70	74	72	–	66	77	89			676
Smith	66	72	–	78	71	74	–	76	65	78	72			652
Pertwee	75	–	79	83	80	78	–	82	72	–	86			635

(a) What is the total amount of prize money won every season? [1]

(b) McGann's win last night was their second win of the season. In which week was their previous win? [1]

(c) There has only been one tie-break so far this season. Which team won it? Explain your reasoning. [3]

(d) Which team has scored 80 points or more four times, but has so far failed to win any prize money? [1]

(e) Hartnell's win last week was achieved with the highest final score by any team this season. They only failed to answer 7 questions correctly.

How many of these 7 questions were picture questions? [2]

(f) Which week's quiz seems to have been the hardest? Give a reason for your answer. [1]

Eccleston now have 795 points because their score of 73 in week 9 has been replaced by last night's 78. McCoy, however, still have 787 points because last night was the lowest of their scores. Troughton have moved up to third place because their previous total was only made up of 9 scores.

(g) (i) List all the teams' league totals as they are now, following last night's quiz. [4]

(ii) Identify one team that cannot win the league, whatever happens in week 13, and explain why not. [2]

8

BLANK PAGE