



# **Cambridge International Examinations**

Cambridge International Advanced Level

CANDIDATE NAME			
CENTRE NUMBER		CANDIDATE NUMBER	
FURTHER MATHEM	IATICS		9231/12
Paper 1			May/June 2018
			3 hours
Candidates answer of	on the Question Paper.		
Additional Materials:	List of Formulae (MF10)		

#### **READ THESE INSTRUCTIONS FIRST**

Write your Centre number, candidate number and name in the spaces at the top of this page.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

DO NOT WRITE IN ANY BARCODES.

Answer **all** the questions in the space provided. If additional space is required, you should use the lined page at the end of this booklet. The question number(s) must be clearly shown.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of a calculator is expected, where appropriate.

Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.



1 The curve C is defined parametrically	יט עו	by
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$x = e^t - t,  y = 4e^{\overline{2}^t}.$
Find the length of the arc of $C$ from the point where $t = 0$ to the point where $t = 3$ . [5]

induction that $f(n)$ is divisible by 9 for every positive integer $n$ .		
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3	The curve C has polar equation $r = \cos 2\theta$ , for $-\frac{1}{4}\pi \le \theta \le \frac{1}{4}\pi$ .	
	(i) Sketch C.	[2]
	(ii) Find the area of the region enclosed by $C$ , showing full working.	[3]
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(iii)	Find a cartesian equation of $C$ .	[3]
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4	It is	given	that	the	equation

$$x^3 - 21x^2 + kx - 216 = 0,$$

where k is a constant, has real roots a, ar and  $ar^{-1}$ .

Find the numerical values of the roots.	[6

(ii)	Deduce the value of $k$ . [2]

$r^2$

(i)	Use the standard result for $\sum_{r=1}^{n} r^2$ given in the List of Formulae (MF10) to show that
	$S_{2n} = -n(2n+1).   [4]$

State the value of $\lim_{n\to\infty} \frac{S_{2n}}{n^2}$ and find $\lim_{n\to\infty} \frac{S_{2n+1}}{n^2}$ .	
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**6** The curve *C* has equation

$$y = \frac{x^2 + b}{x + b},$$

where b is a positive constant.

(i)	Find the equations of the asymptotes of $C$ .	[3]
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(ii)	Show that <i>C</i> does not intersect the <i>x</i> -axis.	[1]

(iii)	Justifying your answer, find the number of stationary points on $C$ . [2]
(iv)	Sketch C. Your sketch should indicate the coordinates of any points of intersection with the y-axis. You do not need to find the coordinates of any stationary points. [3]

12	
Find the particular solution of the differential equation	
$49\frac{d^2y}{dx^2} + 14\frac{dy}{dx} + y = 49x + 735,$	
given that when $x = 0$ , $y = 0$ and $\frac{dy}{dx} = 0$ .	[10]

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8	The linear	transformation	n T:	$\mathbb{R}^4 \to$	$\mathbb{R}^3$ is	represented	by the	matrix M,	where
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$$\mathbf{M} = \begin{pmatrix} 1 & 2 & \alpha & -1 \\ 2 & 6 & -3 & -3 \\ 3 & 10 & -6 & -5 \end{pmatrix}$$

and  $\alpha$  is a constant. When  $\alpha \neq 0$  the null space of T is denoted by  $K_1$ .

(i)	Find a basis for $K_1$ .	[5]

When  $\alpha = 0$  the null space of T is denoted by  $K_2$ .

(ii)	Find a basis for $K_2$ .	[3]
(iii)	Determine, justifying your answer, whether $K_1$ is a subspace of $K_2$ .	[2]

	bstitution $u = \tan x$ , or otherwise, find $\int \sec^2 x \tan^2 x  dx$ .	
•••••		
It is given that, fo	or $n \ge 0$ ,	
	$I_n = \int_0^{\frac{1}{4}\pi} \sec^n x \tan^2 x  \mathrm{d}x.$	
	sult that $\frac{d}{dx}(\sec x) = \tan x \sec x$ , show that, for $n \ge 2$ ,	
(ii) Using the res	$\frac{dx}{dx}(\sec x) = \tan x \sec x, \text{ show that, for } n \ge 2,$	
(ii) Using the res		
(ii) Using the res	$dx^{(see x)} = \tan x \sec x, \text{ show that, for } n \ge 2,$ $(n+1)I_n = (\sqrt{2})^{n-2} + (n-2)I_{n-2}.$	
(ii) Using the res		

ii)	Hence find the mean value of $\sec^4 x \tan^2 x$ with respect to x over the interval $0 \le x \le \frac{1}{4}\pi$ , giving your answer in exact form
i)	Hence find the mean value of $\sec^4 x \tan^2 x$ with respect to $x$ over the interval $0 \le x \le \frac{1}{4}\pi$ , giving your answer in exact form.
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10	The line $l_1$ is parallel to the vector $a\mathbf{i} - \mathbf{j} + \mathbf{k}$ , where $a$ is a constant, and passes through the powhose position vector is $9\mathbf{j} + 2\mathbf{k}$ . The line $l_2$ is parallel to the vector $-a\mathbf{i} + 2\mathbf{j} + 4\mathbf{k}$ and passes through the point whose position vector is $-6\mathbf{i} - 5\mathbf{j} + 10\mathbf{k}$ .							
	( <b>i</b> ) It is	is given that $l_1$ and $l_2$ intersect.						
	(a)	Show that $a = -\frac{6}{13}$ .	[3]					

<b>(b)</b>	Find a cartesian equation of the plane containing $l_1$ and $l_2$ .	[4]
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11 Answer only **one** of the following two alternatives.

## **EITHER**

(i) Show that if $z = e^{i\theta}$ and $z \neq -1$ then						
		$\frac{z-1}{z+1} = i \tan \frac{1}{2}\theta.$	[3]			

$\frac{z^3 - 1}{z^3 + 1} + \frac{z^2 - 1}{z^2 + 1}$	$+\frac{z-1}{z+1}=0.$	[5

<b>43</b>	
i) Hence write down three roots of the equation	
$(z^3 - 1)(z^2 + 1)(z + 1) + (z^2 - 1)(z^3 + 1)(z + 1) + (z - 1)(z^3 + 1)(z^2 + 1) = 0$	
and find the other three roots. Give your answers in an exact form.	[6]
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11 18	given that $e$ is an eigenvector of the matrix $A$ , with corresponding eigenvalue $\lambda$ .	
(i)	Write down another eigenvector of <b>A</b> corresponding to $\lambda$ .	[1]
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		•••••
(ii)	Write down an eigenvector and corresponding eigenvalue of $A^n$ , where $n$ is a positive integration of $A^n$ , where $A^n$ is a positive integration of $A^n$ , where $A^n$ is a positive integration of $A^n$ .	ger. [2]
Let	$\mathbf{A} = \begin{pmatrix} 3 & 0 & 0 \\ 2 & 7 & 0 \\ 4 & 8 & 1 \end{pmatrix}.$	
(iii)	Find a matrix <b>P</b> and a diagonal matrix <b>D</b> such that $A^n = PDP^{-1}$ .	[7]


(iv) Determine the set of values of the real constant k such that

$\sum_{n=1} k^n (\mathbf{A}^n - k\mathbf{A}^{n+1}) = k\mathbf{A}.$	[4]
<i>n</i> =1	

# **Additional Page**

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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