



Cambridge Assessment International Education

Cambridge International Advanced Level

CANDIDATE NAME						
CENTRE NUMBER				ANDIDATE JMBER		
MATHEMATICS	}					9709/31
Paper 3 Pure M	lathematics	3 (P3)		0	ctober/Nov	ember 2019
					1 hour	45 minutes
Candidates ans	wer on the	Question Pa	per.			
Additional Mater	ials: Li	st of Formu	lae (MF9)			

READ THESE INSTRUCTIONS FIRST

Write your centre number, candidate number and name in the spaces at the top of this page.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

DO NOT WRITE IN ANY BARCODES.

Answer **all** the questions in the space provided. If additional space is required, you should use the lined page at the end of this booklet. The question number(s) must be clearly shown.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

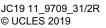
The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 75.

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1

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Solve the inequality $ 2x - 3 > 4 x + 1 $.	[4]

2

The parametric equations o	f a curve are			
	$x = 2t + \sin 2t,$	$y = \ln(1 - \cos \theta)$	(32t).	
Show that $\frac{dy}{dx} = \csc 2t$.				[5
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The number of insects in a population t weeks after the start of observations is denoted by N. The population is decreasing at a rate proportional to $Ne^{-0.02t}$. The variables N and t are treated as continuous, and it is given that when t = 0, N = 1000 and $\frac{dN}{dt} = -10$. (i) Show that N and t satisfy the differential equation $\frac{dN}{dt} = -0.01e^{-0.02t}N.$ [1] (ii) Solve the differential equation and find the value of t when N = 800. [6]

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(iii)	State what happens to the value of N as t becomes large. [1]

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Sh	ow that p satisfies the equation $x = 1 + \exp\left(\frac{1}{2(x-1)}\right)$, where $\exp(x)$ denotes e^x .
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6	(i)	By differentiating	$\frac{\cos x}{\sin x}$, show	that if $y = \cot x$ t	$hen \frac{dy}{dx} = -\csc^2 x$	<i>a</i> . [2]
		-1				
	(ii)	Show that $\int_{\frac{1}{4}\pi}^{\frac{1}{2}\pi} x \cos x$	$\csc^2 x \mathrm{d}x =$	$\frac{1}{4}(\pi + \ln 4).$		[6]
	(ii)	Show that $\int_{\frac{1}{4}\pi}^{\frac{\pi}{2}} x \cos x \cos x$	$\csc^2 x \mathrm{d}x =$	$\frac{1}{4}(\pi + \ln 4).$		[6]
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	(ii)	Show that $\int_{\frac{1}{4}\pi}^{\frac{\pi}{2}} x \cos x \cos x$	$\csc^2 x \mathrm{d}x =$	$\frac{1}{4}(\pi + \ln 4).$		[6]
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When a has this value, find the equation of the plane containing l and m .	[5]
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8	Let $f(x) =$	$x^2 + x + 6$
o	Let $I(x)$ =	$\frac{1}{x^2(x+2)}$

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i) Hence, showing full working, show that the exact value	of $\int_1^{} f(x) dx$ is $\frac{9}{4}$.	[5]
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By first expanding $cos(2x + x)$, show that $cos 3x = 4 cos^3 x - 3 cos x$.	[4]
	10 3
Hence solve the equation $\cos 3x + 3\cos x + 1 = 0$, for $0 \le x \le \pi$.	[2]

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···	Find the exact value of $\int_{\frac{1}{6}\pi}^{\frac{1}{3}\pi} \cos^3 x dx.$ [4]
(111)	Find the exact value of $\int_{1}^{3} \cos^{3} x dx$. [4]
	$m{J}_{ar{G}}^{m{\pi}}$

10	(a)	The complex number u is given by $u = -3 - (2\sqrt{10})i$. Showing all necessary working and without using a calculator, find the square roots of u . Give your answers in the form $a + ib$, where the numbers a and b are real and exact. [5]

(b) On a sketch of an Argand diagram shade the region whose points represent complex numbers z satisfying the inequalities $|z-3-\mathrm{i}| \le 3$, $\arg z \ge \frac{1}{4}\pi$ and $\operatorname{Im} z \ge 2$, where $\operatorname{Im} z$ denotes the imaginary part of the complex number z.

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