Cambridge Assessment International Education

# Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER | CANDIDATE NUMBER

*7729284446*

**FURTHER MATHEMATICS** **9231/22**

Paper 2 Further Pure Mathematics 2 **October/November 2020**

**2 hours**

You must answer on the question paper.

You will need: List of formulae (MF19)

**INSTRUCTIONS**

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

**INFORMATION**

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Blank pages are indicated.

1 Find the Maclaurin's series for $\tan\left(x+\frac{1}{4}\pi\right)$ up to and including the term in $x^2$. [5]

2 A curve has equation $y = \cosh x$, for $0 \leqslant x \leqslant \frac{1}{2}$.

Find, in terms of $\pi$ and e, the area of the surface generated when the curve is rotated through $2\pi$ radians about the $x$-axis. [6]

3 Find all the roots of the equation $(w+1)^6 = 1$, giving your answers in the form $x + \mathrm{i}y$ where $x$ and $y$ are real and exact. [4]

**4** Find the solution of the differential equation

$$x\frac{dy}{dx} + 2y = e^x$$

for which $y = 3$ when $x = 1$. Give your answer in the form $y = f(x)$. [8]

**5** The curve $C$ has equation

$$y^2 + (xy+1)^2 = 5.$$

**(a)** Show that, at the point $(1,1)$ on $C$, $\frac{dy}{dx} = -\frac{2}{3}$. [3]

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**(b)** Find the value of $\frac{d^2y}{dx^2}$ at the point $(1,1)$. [5]

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**6** Find the particular solution of the differential equation

$$\frac{d^2x}{dt^2} + 8\frac{dx}{dt} + 15x = 102\cos 3t,$$

given that, when $t = 0$, $x = 1$ and $\frac{dx}{dt} = 0$. [11]

 

**7** **(a)** Show that $\sum_{r=1}^{n} z^{2r} = \frac{z^{2n+1} - z}{z - z^{-1}}$, for $z \neq 0, 1, -1$. [2]

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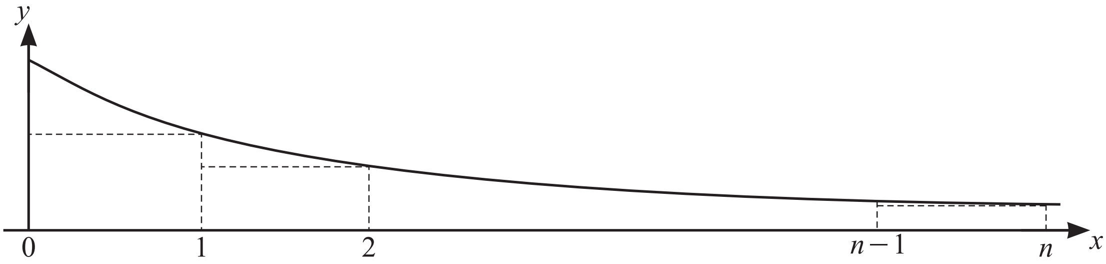
**(b)** By letting $z = \cos\theta + \mathrm{i}\sin\theta$, show that, if $\sin\theta \neq 0$,

$$1 + 2\sum_{r=1}^{n} \cos(2r\theta) = \frac{\sin(2n+1)\theta}{\sin\theta}.$$ [5]

**8**




The diagram shows the curve $y = \dfrac{1}{\sqrt{x^2+x+1}}$ for $x \geqslant 0$, together with a set of $n$ rectangles of unit width. By considering the sum of the areas of these rectangles, show that

$$\sum_{r=1}^{n} \frac{1}{\sqrt{r^2+r+1}} < \ln\left(\tfrac{1}{3} + \tfrac{2}{3}n + \tfrac{2}{3}\sqrt{n^2+n+1}\right).$$ [10]

9 It is given that $a$ is a positive constant.

**(a)** Show that the system of equations

$$\begin{aligned} ax + (2a+5)y + (a+1)z &= 1, \\ -4y \quad &= 2, \\ 3y - z &= 3, \end{aligned}$$

has a unique solution and interpret this situation geometrically. [3]

The matrix **A** is given by

$$\mathbf{A} = \begin{pmatrix} a & 2a+5 & a+1 \\ 0 & -4 & 0 \\ 0 & 3 & -1 \end{pmatrix}.$$

**(b)** Show that the eigenvalues of **A** are $a$, $-1$ and $-4$. [2]

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**(c)** Find a matrix **P** such that

$$\mathbf{A} = \mathbf{P}\begin{pmatrix} a & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -4 \end{pmatrix}\mathbf{P}^{-1}.$$ [5]

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**(d)** Use the characteristic equation of **A** to find $\mathbf{A}^{-1}$. [6]

## Additional Page

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