

# Cambridge International AS & A Level

CANDIDATE NAME					
CENTRE NUMBER			CANDIDATE NUMBER		

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#### **FURTHER MATHEMATICS**

9231/32

Paper 3 Further Mechanics

October/November 2020

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

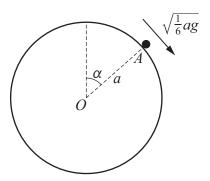
#### **INSTRUCTIONS**

- Answer all questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do not write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity (g) is needed, use  $10 \,\mathrm{m\,s^{-2}}$ .

#### **INFORMATION**

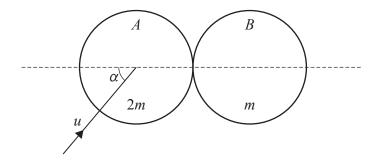
- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

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A fixed smooth solid sphere has centre O and radius a. A particle of mass m is projected downwards with speed  $\sqrt{\frac{1}{6}ag}$  from the point A on the surface of the sphere, where OA makes an angle  $\alpha$  with the upward vertical through O (see diagram). The particle moves in part of a vertical circle on the surface of the sphere. It loses contact with the sphere at the point B, where OB makes an angle  $\beta$  with the upward vertical through O.

Given that $\cos \alpha = \frac{2}{3}$ , find the value of $\cos \beta$ .	[5]
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Two uniform smooth spheres A and B of equal radii have masses 2m and m respectively. Sphere B is at rest on a smooth horizontal surface. Sphere A is moving on the surface with speed u and collides with B. Immediately before the collision, the direction of motion of A makes an angle  $\alpha$  with the line of centres of the spheres, where  $\tan \alpha = \frac{4}{3}$  (see diagram). The coefficient of restitution between the spheres is  $\frac{1}{3}$ .

Find the speed of A after the collision.	[5]

man	erial.
(a)	Find the distance of the centre of mass of the object from the end of the cylinder that is not attact to the cone.

norizontal surface.	[3]

$\cos\theta = \frac{g}{\omega^2 r}.$	

The plane of the circular motion is at a height x above the lowest point of the shell. When the angular speed is doubled, the plane of the motion is at a height 4x above the lowest point of the shell.

Find $x$ in terms of $r$ .	[4
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A particle P is projected with speed  $u \, \mathrm{m} \, \mathrm{s}^{-1}$  at an angle of  $\theta$  above the horizontal from a point O on a horizontal plane and moves freely under gravity. The horizontal and vertical displacements of P from O at a subsequent time ts are denoted by x m and y m respectively. (a) Starting from the equation of the trajectory given in the List of formulae (MF19), show that  $y = x \tan \theta - \frac{gx^2}{2u^2} (1 + \tan^2 \theta).$ [1] When  $\theta = \tan^{-1} 2$ , P passes through the point with coordinates (10, 16). (b) Show that there is no value of  $\theta$  for which P can pass through the point with coordinates (18, 30). [6]

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speed $\sqrt{2ga}$ . At this point, its acceleration is $\frac{7}{3}g$ downwards.							
Show that $k = 4mg$ and find in terms of a the greatest height above Q reached by F	P. [8]						


	$nat x = \frac{1}{k} \ln 2$	- ,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,	2				
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Beginning at the instant when the speed of P is  $\frac{1}{2}u$ , an additional force acts on P. This force has magnitude  $\frac{5m}{v}$ N and acts in the direction of increasing x.

(b)	Show that when the speed of $P$ has increased again to $u  \text{m s}^{-1}$ , the total distance travelled by $P$ is given by an expression of the form
	$\frac{1}{3k}\ln\left(\frac{A-ku^3}{B-ku^3}\right),$

stating the values of the constants $A$ and $B$ .	[7]

# **Additional Page**

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