

# Cambridge International AS & A Level

CANDIDATE NAME					
CENTRE NUMBER			CANDIDATE NUMBER		

# 993642171

### **FURTHER MATHEMATICS**

9231/11

Paper 1 Further Pure Mathematics 1

May/June 2021

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

### **INSTRUCTIONS**

- Answer all questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do not use an erasable pen or correction fluid.
- Do not write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

### **INFORMATION**

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has 16 pages. Any blank pages are indicated.

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(	b)	Show	that
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**(c)** 

$\underline{\hspace{1cm} 1-r-r^2} \underline{\hspace{1cm}} \underline{\hspace{1cm}} r+1 \underline{\hspace{1cm}} \underline{\hspace{1cm}} r$
$\frac{1-r-r^2}{(r^2+2r+2)(r^2+1)} = \frac{r+1}{(r+1)^2+1} - \frac{r}{r^2+1}$
$1-r-r^2$
and hence use the method of differences to find $\sum_{r=1}^{n} \frac{1-r-r^2}{\left(r^2+2r+2\right)\left(r^2+1\right)}.$ [5]
7-1 (
Deduce the value of $\sum_{r=1}^{\infty} \frac{1-r-r^2}{(r^2+2r+2)(r^2+1)}$ . [1]
$r=1$ $(r^2+2r+2)(r^2+1)$

J	Find a quartic equation whose roots are $\alpha^3$ , $\beta^3$ , $\gamma^3$ , $\delta^3$ and state the value of $\alpha^3 + \beta^3 + \gamma$
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	the value of $\frac{1}{\alpha^3} + \frac{1}{\beta^3} + \frac{1}{\gamma^3} + \frac{1}{\delta^3}$ .
	411
	the value of $\alpha^4 + \beta^4 + \gamma^4 + \delta^4$ .

` /	Find $\mathbf{M}$ in terms of $d$ .	
(b)	The unit square in the $x$ - $y$ plane is transformed by $\mathbf{M}$ onto a parallelogram of area	$\frac{1}{2}d^2$ units <sup>2</sup> .
(b)	The unit square in the <i>x-y</i> plane is transformed by $\mathbf{M}$ onto a parallelogram of area Show that $d=2$ .	$\frac{1}{2}d^2$ units <sup>2</sup> .
<b>(b)</b>		$\frac{1}{2}d^2$ units <sup>2</sup> .
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<b>(b)</b>		$\frac{1}{2}d^2$ units <sup>2</sup> .
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(b)		

The	e matrix <b>N</b> is such that $\mathbf{MN} = \begin{pmatrix} 1 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$ .	
	Find N.	[3]
(d)	Find the equations of the invariant lines represented by <b>MN</b> .	s, through the origin, of the transformation in the $x$ - $y$ plane [5]

5	The	curve C has polar equation $r = a \cot(\frac{1}{3}\pi - \theta)$ , where a is a positive constant and $0 \le \theta \le \frac{1}{6}\pi$ .	
		given that the greatest distance of a point on C from the pole is $2\sqrt{3}$ .	
	(a)	Sketch $C$ and show that $a = 2$ .	[3]
			•••••
	(b)	Find the exact value of the area of the region bounded by $C$ , the initial line and the half- $\theta = \frac{1}{6}\pi$ .	-line [4]

(c)	Show that C has Cartesian equation $2(x+y\sqrt{3}) = (x\sqrt{3}-y)\sqrt{x^2+y^2}$ . [3]

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6	Let t be	a positive	constant.

The line  $l_1$  passes through the point with position vector  $t\mathbf{i} + \mathbf{j}$  and is parallel to the vector  $-2\mathbf{i} - \mathbf{j}$ . The line  $l_2$  passes through the point with position vector  $\mathbf{j} + t\mathbf{k}$  and is parallel to the vector  $-2\mathbf{j} + \mathbf{k}$ .

It is given that the shortest distance between the lines  $l_1$  and  $l_2$  is  $\sqrt{21}$ .

(a)	Find the value of <i>t</i> .	[5]
The	plane $\Pi_1$ contains $l_1$ and is parallel to $l_2$ .	
	Write down an equation of $\Pi_1$ , giving your answer in the form $\mathbf{r} = \mathbf{a} + \lambda \mathbf{b} + \mu \mathbf{c}$ .	[1]

The plane  $\Pi_2$  has Cartesian equation 5x - 6y + 7z = 0. (c) Find the acute angle between  $l_2$  and  $\Pi_2$ . [3] (d) Find the acute angle between  $\Pi_1$  and  $\Pi_2$ . [3] .....

a)	Find the equations of the asymptotes of <i>C</i> .	
	Find the coordinates of the stationary points on C	
))	Find the coordinates of the stationary points on $C$ .	
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<b>)</b> )	Find the coordinates of the stationary points on <i>C</i> .	
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(c) Sketch C, stating the coordinates of any intersections with the axes.	[3]
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(d) Sketch the curve with equation  $y = \left| \frac{x^2 + x + 9}{x + 1} \right|$  and find the set of values of x for which

 $2|x^2 + x + 9| > 13|x + 1|.$  [5]

## **Additional Page**

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.						
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