

# Cambridge International AS & A Level

CANDIDATE NAME					
CENTRE NUMBER			CANDIDATE NUMBER		

# 6529416975

## **FURTHER MATHEMATICS**

9231/23

Paper 2 Further Pure Mathematics 2

May/June 2021

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

### **INSTRUCTIONS**

- Answer all questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do not write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

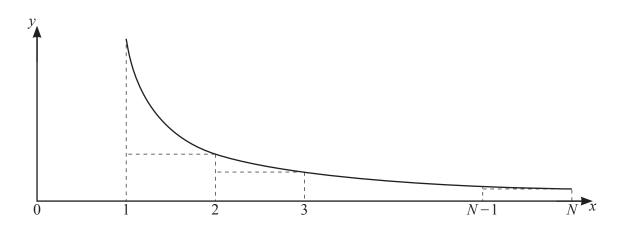
### **INFORMATION**

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has 16 pages.

(a)	Find a and b such that	
	$z^8 - iz^5 - z^3 + i = (z^5 - a)(z^3 - b).$	[
(b)	Hence find the roots of	
	$z^8 - iz^5 - z^3 + i = 0,$	
	giving your answers in the form $re^{i\theta}$ , where $r > 0$ and $0 \le \theta \le 2\pi$ .	[


3



The diagram shows the curve  $y = \frac{x}{2x^2 - 1}$  for  $x \ge 1$ , together with a set of N - 1 rectangles of unit width.

(a)	By considering	the sum	of the	areas of these	rectangles	chow	that
(a)	By considering	me sum	or me	areas or mese	rectangles.	SHOW	Шa

$\sum_{r=1}^{\infty} \frac{1}{2r^2 - 1}$	$\frac{1}{4}$ In $(2N-1)$	<b>†</b> 1.	[/]
r=1			

(b)	Use a similar method to find, in terms of $N$ , a lower bound for $\sum_{r=1}^{N} \frac{r}{2r^2 - 1}$ . [3]

$\tan^5 \theta = \frac{\sin}{\cos}$	$\frac{n 5\theta - a \sin 3\theta + b \sin \theta}{s 5\theta + a \cos 3\theta + b \cos \theta},$
where $a$ and $b$ are integers to be determined	d. [7]

5 The variables x and y are related by the different	ntial e	equation
--	---------	----------

by the differential equation 
$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 3y = 4e^{-x}.$$

,	Find the value of the constant $k$ such that $y = kxe^{-x}$ is a particular integral of the different equation.	4
		•••
		•••
		•••
		• • •
		•••
	Find the solution of the differential equation for which $y = \frac{dy}{dx} = \frac{1}{2}$ when $x = 0$ .	6
		•••
		•••
		• • •

	$2\sinh^2 x = \cosh 2x - 1.$	[3]
		•••••
(b)	Find the solution to the differential equation	
(b)	Find the solution to the differential equation $\frac{dy}{dx} + y \coth x = 4 \sinh x$	
(b)	$\frac{\mathrm{d}y}{\mathrm{d}x} + y \coth x = 4 \sinh x$	[7]
(b)		[7]
(b)	$\frac{\mathrm{d}y}{\mathrm{d}x} + y \coth x = 4 \sinh x$	[7]
(b)	$\frac{\mathrm{d}y}{\mathrm{d}x} + y \coth x = 4 \sinh x$	[7]
(b)	$\frac{\mathrm{d}y}{\mathrm{d}x} + y \coth x = 4 \sinh x$	
(b)	$\frac{dy}{dx} + y \coth x = 4 \sinh x$ for which $y = 1$ when $x = \ln 3$ .	
(b)	$\frac{dy}{dx} + y \coth x = 4 \sinh x$ for which $y = 1$ when $x = \ln 3$ .	
(b)	$\frac{dy}{dx} + y \coth x = 4 \sinh x$ for which $y = 1$ when $x = \ln 3$ .	
(b)	$\frac{dy}{dx} + y \coth x = 4 \sinh x$ for which $y = 1$ when $x = \ln 3$ .	
(b)	$\frac{dy}{dx} + y \coth x = 4 \sinh x$ for which $y = 1$ when $x = \ln 3$ .	
(b)	$\frac{dy}{dx} + y \coth x = 4 \sinh x$ for which $y = 1$ when $x = \ln 3$ .	
(b)	$\frac{dy}{dx} + y \coth x = 4 \sinh x$ for which $y = 1$ when $x = \ln 3$ .	
(b)	$\frac{dy}{dx} + y \coth x = 4 \sinh x$ for which $y = 1$ when $x = \ln 3$ .	
(b)	$\frac{dy}{dx} + y \coth x = 4 \sinh x$ for which $y = 1$ when $x = \ln 3$ .	
(b)	$\frac{dy}{dx} + y \coth x = 4 \sinh x$ for which $y = 1$ when $x = \ln 3$ .	
(b)	$\frac{dy}{dx} + y \coth x = 4 \sinh x$ for which $y = 1$ when $x = \ln 3$ .	

a)	Find the exact value of $I_1$ , expressing your answer in logarithmic form.	
		••••••
(b)	By considering $\frac{d}{dx} \left( x \left( 4 + x^2 \right)^{-\frac{1}{2}n} \right)$ , or otherwise, show that $4nI_{n+2} = \frac{3}{2} \left( \frac{2}{5} \right)^n + (n-1)I_n.$	
(b)	By considering $\frac{d}{dx}(x(4+x^2)^{-\frac{1}{2}n})$ , or otherwise, show that $4nI_{n+2} = \frac{3}{2}(\frac{2}{5})^n + (n-1)I_n.$	
<b>(b)</b>		
(b)		
b)		
(b)	$4nI_{n+2} = \frac{3}{2} \left(\frac{2}{5}\right)^n + (n-1)I_n.$	
b)	$4nI_{n+2} = \frac{3}{2} \left(\frac{2}{5}\right)^n + (n-1)I_n.$	
(b)	$4nI_{n+2} = \frac{3}{2} \left(\frac{2}{5}\right)^n + (n-1)I_n.$	
(b)	$4nI_{n+2} = \frac{3}{2} \left(\frac{2}{5}\right)^n + (n-1)I_n.$	
(b)	$4nI_{n+2} = \frac{3}{2} \left(\frac{2}{5}\right)^n + (n-1)I_n.$	
(b)	$4nI_{n+2} = \frac{3}{2} \left(\frac{2}{5}\right)^n + (n-1)I_n.$	
(b)	$4nI_{n+2} = \frac{3}{2} \left(\frac{2}{5}\right)^n + (n-1)I_n.$	
(b)	$4nI_{n+2} = \frac{3}{2} \left(\frac{2}{5}\right)^n + (n-1)I_n.$	
(b)	$4nI_{n+2} = \frac{3}{2} \left(\frac{2}{5}\right)^n + (n-1)I_n.$	

		••
		• •
		••
		••
		• •
		• •
		• •
		• •
		••
		••
		••
		• •
		••
(c)	Find the value of I	
(c)	Find the value of $I_5$ .	  3]
(c)	Find the value of $I_5$ .	  3]
(c)	Find the value of $I_5$ .	
(c)	Find the value of $I_5$ .	 
(c)	Find the value of $I_5$ .	  
(c)		••
(c)	Find the value of $I_5$ .	••
(c)		••
(c)		

(a)	Find the value of a for which the system of equations	
	13x + 18y - 28z = 0, $-4x - ay + 8z = 0,$ $2x + 6y - 5z = 0,$	
	does not have a unique solution.	[2
The	e matrix A is given by	
	$\mathbf{A} = \begin{pmatrix} 13 & 18 & -28 \\ -4 & -1 & 8 \\ 2 & 6 & -5 \end{pmatrix}.$	
(b)	Find the eigenvalue of <b>A</b> corresponding to the eigenvector $\begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$ .	[1
(c)	Find a matrix <b>P</b> and a diagonal matrix <b>D</b> such that $\mathbf{A} = \mathbf{P}\mathbf{D}\mathbf{P}^{-1}$ .	[8

		· • • •
Use the characteristic equation of A	${f A}$ to find ${f A}^{-1}$ in terms of ${f A}$ .	
Use the characteristic equation of A	$\mathbf{A}$ to find $\mathbf{A}^{-1}$ in terms of $\mathbf{A}$ .	[2]
Use the characteristic equation of A	$\mathbf{A}$ to find $\mathbf{A}^{-1}$ in terms of $\mathbf{A}$ .	[2]

(d)

# **Additional Page**

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.			

Permission to reproduce items where third-party owned material protected by copyright is included has been sought and cleared where possible. Every reasonable effort has been made by the publisher (UCLES) to trace copyright holders, but if any items requiring clearance have unwittingly been included, the publisher will be pleased to make amends at the earliest possible opportunity.

To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced online in the Cambridge Assessment International Education Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download at www.cambridgeinternational.org after the live examination series.

Cambridge Assessment International Education is part of the Cambridge Assessment Group. Cambridge Assessment is the brand name of the University of Cambridge Local Examinations Syndicate (UCLES), which itself is a department of the University of Cambridge.