



Cambridge International AS & A Level

CANDIDATE
NAME

--	--	--	--	--

CENTRE
NUMBER

--	--	--	--	--

CANDIDATE
NUMBER

--	--	--	--



FURTHER MATHEMATICS

9231/11

Paper 1 Further Pure Mathematics 1

October/November 2021

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Any blank pages are indicated.

- 1 It is given that

$$\alpha + \beta + \gamma = 3, \quad \alpha^2 + \beta^2 + \gamma^2 = 5, \quad \alpha^3 + \beta^3 + \gamma^3 = 6.$$

The cubic equation $x^3 + bx^2 + cx + d = 0$ has roots α, β, γ .

Find the values of b, c and d .

[6]

- 2 (a) Use standard results from the list of formulae (MF19) to find $\sum_{r=1}^n r(r+1)(r+2)$ in terms of n , fully factorising your answer. [3]

- (b) Express $\frac{1}{r(r+1)(r+2)}$ in partial fractions and hence use the method of differences to find

$$\sum_{r=1}^n \frac{1}{r(r+1)(r+2)}. \quad [5]$$

- (c) Deduce the value of $\sum_{r=1}^{\infty} \frac{1}{r(r+1)(r+2)}$. [1]

- 3 The sequence of real numbers a_1, a_2, a_3, \dots is such that $a_1 = 1$ and

$$a_{n+1} = \left(a_n + \frac{1}{a_n} \right)^3.$$

- (a) Prove by mathematical induction that $\ln a_n \geq 3^{n-1} \ln 2$ for all integers $n \geq 2$. [6]

[You may use the fact that $\ln\left(x + \frac{1}{x}\right) > \ln x$ for $x > 0$.]

.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....

- (b) Show that $\ln a_{n+1} - \ln a_n > 3^{n-1} \ln 4$ for $n \geq 2$. [2]

.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....

- 4 The matrix \mathbf{M} is given by $\mathbf{M} = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix}$.

- (a) The matrix \mathbf{M} represents a sequence of two geometrical transformations.

State the type of each transformation, and make clear the order in which they are applied. [2]

.....

- (b) Find the values of θ , for $0 \leq \theta \leq \pi$, for which the transformation represented by \mathbf{M} has exactly one invariant line through the origin, giving your answers in terms of π . [9]

.....

.....

.....

.....

.....

- 5 The plane Π has equation $\mathbf{r} = -2\mathbf{i} + 3\mathbf{j} + 3\mathbf{k} + \lambda(\mathbf{i} + \mathbf{k}) + \mu(2\mathbf{i} + 3\mathbf{j})$.

(a) Find a Cartesian equation of Π , giving your answer in the form $ax + by + cz = d$.

[4]

The line l passes through the point P with position vector $2\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}$ and is parallel to the vector \mathbf{k} .

(b) Find the position vector of the point where l meets Π .

[3]

- (c) Find the acute angle between l and Π .

[3]

.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....

- (d) Find the perpendicular distance from P to Π .

[3]

.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....

- 6** The curve C has polar equation $r = 2 \cos \theta(1 + \sin \theta)$, for $0 \leq \theta \leq \frac{1}{2}\pi$.

(a) Find the polar coordinates of the point on C that is furthest from the pole. [5]

(b) Sketch C .

[2]

(c) Find the area of the region bounded by C and the initial line, giving your answer in exact form. [6]

.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....

- 7 The curve C has equation $y = \frac{4x+5}{4-4x^2}$.

(a) Find the equations of the asymptotes of C .

[2]

.....
.....
.....
.....
.....
.....
.....
.....

(b) Find the coordinates of any stationary points on C .

[4]

.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....

(c) Sketch C , stating the coordinates of the intersections with the axes.

[3]

(d) Sketch the curve with equation $y = \left| \frac{4x+5}{4-4x^2} \right|$ and find in exact form the set of values of x for which $4|4x+5| > 5|4-4x^2|$.

[6]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

BLANK PAGE

Permission to reproduce items where third-party owned material protected by copyright is included has been sought and cleared where possible. Every reasonable effort has been made by the publisher (UCLES) to trace copyright holders, but if any items requiring clearance have unwittingly been included, the publisher will be pleased to make amends at the earliest possible opportunity.

To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced online in the Cambridge Assessment International Education Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download at www.cambridgeinternational.org after the live examination series.

Cambridge Assessment International Education is part of the Cambridge Assessment Group. Cambridge Assessment is the brand name of the University of Cambridge Local Examinations Syndicate (UCLES), which itself is a department of the University of Cambridge.