

Cambridge International AS & A Level

CANDIDATE NAME						
CENTRE NUMBER				CANDIDATE NUMBER		

FURTHER MATHEMATICS

9231/12

Paper 1 Further Pure Mathematics 1

October/November 2021

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer all questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do not write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has 16 pages. Any blank pages are indicated.

(a)	Give full details of the geometrical transformation in the <i>x-y</i> plane represented by the matrix $\begin{pmatrix} 6 & 0 \\ 0 & 6 \end{pmatrix}$.	trix [1]
Let	$\mathbf{A} = \begin{pmatrix} 3 & 4 \\ 2 & 2 \end{pmatrix}.$	•••••
(b)	The triangle DEF in the x-y plane is transformed by A onto triangle PQR .	
	Given that the area of triangle <i>DEF</i> is 13 cm ² , find the area of triangle <i>PQR</i> .	[2]
(c)	Find the matrix B such that $\mathbf{AB} = \begin{pmatrix} 6 & 0 \\ 0 & 6 \end{pmatrix}$.	[2]
(d)	Show that the origin is the only invariant point of the transformation in the x - y plane represer by \mathbf{A} .	nted [4]

Prove by mathematical ir	duction that, for all positive integer	
	$\frac{\mathrm{d}^n y}{\mathrm{d} x^n} = \left(a^n x + n a^{n-1} \right) \mathrm{e}^{ax}.$	[6
	$\mathbf{d}x$	
•••••		

2	Let $S_{-} =$	$\sum_{n=1}^{n}$	r(r+2)
J	Let S_n –	· 🚄 '''	$(r+1)^2$
		$\nu=1$	(r+1)

				, show th	n	2(n+1)		
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Let
$$S = \sum_{r=1}^{\infty} \ln \frac{r(r+2)}{(r+1)^2}$$
.

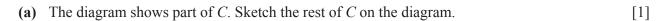
Find the least value of <i>n</i> such that $S_n - S < 0.01$.	
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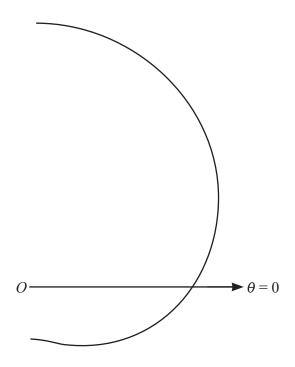
(a)	Find the value of $\alpha^2 + \beta^2 + \gamma^2$.	[2]
(b)	Show that $\alpha^3 + \beta^3 + \gamma^3 = 1$.	[2]

(c) Use standard results from the list of formulae (MF19) to show that

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5 The curve C has polar equation $r = 3 + 2\sin\theta$, for $-\pi < \theta \le \pi$.





The straight line *l* has polar equation $r \sin \theta = 2$.

Add l to the diagram in part (a) and find the polar coordinates of the points of intersection of and l .

The region R is enclosed by C and I , and contains the pole.	
Find the area of R , giving your answer in exact form.	[6
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	curve C has equation $y = \frac{x^2}{x-3}$.	
	Find the equations of the asymptotes of C .	[
(b)	Show that there is no point on C for which $0 < y < 12$.	[

(c) Sketch *C*. [2]

(d) (i) Sketch the graphs of $y = \left| \frac{x^2}{x-3} \right|$ and y = |x| - 3 on a single diagram, stating the coordinates of the intersections with the axes. [4]

(ii) Use your sketch to find the set of values of c for which $\left|\frac{x^2}{x-3}\right| \le |x| + c$ has no solution. [1]

	7	The	points	A.	В.	C hav	e position	vectors
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$$2\mathbf{i} + 2\mathbf{j}$$
, $-\mathbf{j} + \mathbf{k}$ and $2\mathbf{i} + \mathbf{j} - 7\mathbf{k}$

respectively, relative to the origin O.

	ation of the plane <i>O</i>	DAB, giving your answer	in the form $\mathbf{r.n} = p$.	
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	s equation $x - 3y - 2$			
Find the ne	rpendicular distance	e of Π from the origin.		
i ma me pe				•••••

Find the acute angle between the planes OAB and Π .	
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Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.							
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