

# Cambridge International AS & A Level

CANDIDATE NAME					
CENTRE NUMBER			CANDIDATE NUMBER		

# 8890732124

### **FURTHER MATHEMATICS**

9231/13

Paper 1 Further Pure Mathematics 1

October/November 2021

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

### **INSTRUCTIONS**

- Answer all questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do not write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

### **INFORMATION**

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has 16 pages. Any blank pages are indicated.

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$\alpha + \beta + \gamma = 3$ , $\alpha^2 + \beta^2 + \gamma^2 = 5$ , $\alpha^3 + \beta^3 + \gamma^3 = 6$ .	
The cubic equation $x^3 + bx^2 + cx + d = 0$ has roots $\alpha$ , $\beta$ , $\gamma$ .	
Find the values of $b$ , $c$ and $d$ .	[6]
	,
	,

(a)	Use standard results from the list of formulae (MF19) to find $\sum_{r=1}^{n} r(r+1)(r+2)$ in terms of fully factorising your answer.
(4)	fully factorising your answer. [2]

	$\sum_{r=1}^{n} \frac{1}{r(r+1)(r+2)}.$	[:
Deduce the value of $\sum_{r=1}^{\infty} \frac{1}{r(r+1)(r+1)}$	$\overline{r+2)}$ .	[

3	The sequence o	f real numbers	$a_1, a_2$	$a_1, \dots i$	s such that a	L = 1 and
•	The bequeince o	1 I cai mamoris	$\alpha_1, \alpha_2,$	<i>u</i> <sub>3</sub> , 1	o ouch that a	1 4114

$$a_{n+1} = \left(a_n + \frac{1}{a_n}\right)^3.$$

(a)	Prove by mathematical induction that $\ln a_n \ge 3^{n-1} \ln 2$ for all integers $n \ge 2$ .	5]
	[You may use the fact that $\ln\left(x+\frac{1}{x}\right) > \ln x$ for $x > 0$ .]	
		· <b>-</b>
		•
(b)	Show that $\ln a_{n+1} - \ln a_n > 3^{n-1} \ln 4$ for $n \ge 2$ .	2]

4	The matrix $\mathbf{M}$ is given by $\mathbf{M} =$	$ \cos \theta \\ \sin \theta $	$-\sin\theta \cos\theta$	3	$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

(a)	The matrix <b>M</b> represents a sequence of two geometrical transformations.
	State the type of each transformation, and make clear the order in which they are applied. [2]
(b)	Find the values of $\theta$ , for $0 \le \theta \le \pi$ , for which the transformation represented by <b>M</b> has exactly one invariant line through the origin, giving your answers in terms of $\pi$ . [9]

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Fi	and a Cartesian equation of $\Pi$ , giving your answer in the form $ax + by + cz = d$ .	[4]
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in	the <i>l</i> passes through the point <i>P</i> with position vector $2\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}$ and is parallel to the	vector <b>k</b> .
	the $l$ passes through the point $P$ with position vector $2\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}$ and is parallel to the sind the position vector of the point where $l$ meets $\Pi$ .	vector <b>k</b> .
Fi 		[3]
Fi 	and the position vector of the point where $l$ meets $\Pi$ .	[3]
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F <b>i</b>	and the position vector of the point where $l$ meets $\Pi$ .	[3]
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Find the perpen	dicular distance	e from $P$ to $\Pi$ .	 	
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	dicular distance	e from P to II.		

The	e curve C has polar equation $r = 2\cos\theta(1+\sin\theta)$ , for $0 \le \theta \le \frac{1}{2}\pi$ .	
(a)	Find the polar coordinates of the point on $C$ that is furthest from the pole.	[5]
		,

<b>(b)</b>	Sketch C.	[2]
(c)	Find the area of the region bounded by <i>C</i> and the initial line, giving your answer in exact form.	[6]
(0)	That the treat of the region boanded by a that the initial line, giving your this wor in exact form.	[0]
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7	The curve C has equation w	-4x+5
/	The curve $C$ has equation $y =$	$\frac{1}{4-4r^2}$

(a)	Find the equations of the asymptotes of <i>C</i> .	[2]
(b)	Find the coordinates of any stationary points on <i>C</i> .	[4]

(c)	Sketch <i>C</i> , stating the coordinates of the intersections with the axes.	[3]
(d)	Sketch the curve with equation $y = \left  \frac{4x+5}{4-4x^2} \right $ and find in exact form the set of values of x which $4 4x+5  > 5 4-4x^2 $ .	for [6]

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## **Additional Page**

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