

Cambridge International AS & A Level

CANDIDATE NAME					
CENTRE NUMBER			CANDIDATE NUMBER		

FURTHER MATHEMATICS

9231/21

Paper 2 Further Pure Mathematics 2

October/November 2021

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer all questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do not use an erasable pen or correction fluid.
- Do not write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has 16 pages.

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2 The matrix **A** is given by

$$\mathbf{A} = \begin{pmatrix} -1 & 2 & 12 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{pmatrix}.$$

Use the characteristic equation of A to show that

$$\mathbf{A}^4 = p\mathbf{A}^2 + q\mathbf{I},$$

where p and q are integers to be determined.	[6]

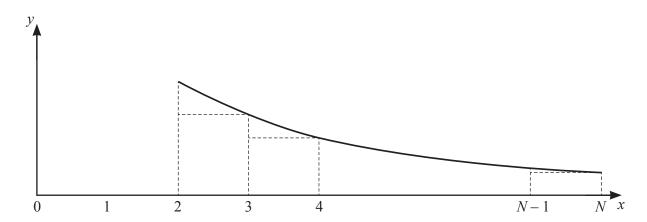
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$$xy^3 - 4x^3y = 3.$$

Show that, at the point $(-1,1)$ on C , $\frac{dy}{dx} = 11$.	

Find the value of $\frac{d^2y}{dx^2}$ at the point (-1,				
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4



The diagram shows the curve with equation $y = \frac{\ln x}{x^2}$ for $x \ge 2$, together with a set of (N-2) rectangles of unit width.

(a) By considering the sum of the areas of these rectangles, show that

$\sum_{r=1}^{N}$	$\frac{\ln r}{r^2} < \frac{2+3\ln 2}{4}$	$-\frac{1+\ln N}{N}.$	[7]

(b)	Use a similar method to find, in terms of N , a lower bound for	$\sum_{r=1}^{N} \frac{\ln r}{r^2}.$	[3]	
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5 Find the particular solution of the differential equation

	$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} - 2\frac{\mathrm{d}y}{\mathrm{d}x} + y = 4\cos x,$	
given that, when $x = 0$, $y = -4$ and	$\frac{\mathrm{d}y}{\mathrm{d}x} = 3.$	1]
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6 (a) Use de Moivre's theorem to show that

$\csc 5\theta =$	$= \frac{\csc^5 \theta}{5 \csc^4 \theta - 20 \csc^2 \theta + 16}.$	[6]

((b)	Hence	obtain	the	roots	of t	he e	quatio	n
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5	10.4	$+40x^{2}$	22	_ 0
x^{-}	I Ux	$+40x^{-}$	-32	= ()

in the form $\csc(q\pi)$, where q is rational.	[4]

7	(a)	Show that an appropriate integrating factor for
		$\sqrt{x^2 - 1} \frac{dy}{dx} + y = x^2 - x\sqrt{x^2 - 1}$
		is $x + \sqrt{x^2 - 1}$. [4]

$\sqrt{x^2 - 1} \frac{dy}{dx} + y = x^2 - x\sqrt{x^2 - 1}$
for which $y = 1$ when $x = \frac{5}{4}$. Give your answer in the form $y = f(x)$.

	2	
	$2\cosh^2 A = \cosh 2A + 1.$	[3]
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The cur	C has parametric equations	
	$x = 2\cosh 2t + 3t$, $y = \frac{3}{2}\cosh 2t - 4t$, for $-\frac{1}{2} \le t \le \frac{1}{2}$.	
The one		ovia ia domotod
by A .	ea of the surface generated when C is rotated through 2π radians about the	y-axis is denoted
-	Show that $A = 10\pi \int_{-\frac{1}{2}}^{\frac{1}{2}} (2\cosh 2t + 3t)\cosh 2t dt$.	F 43
(b) (1)	Show that $A = 10\pi \left((2 \cosh 2t + 3t) \cosh 2t \right) dt$.	[4]
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Additional Page

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