

Cambridge International AS & A Level

CANDIDATE NAME					
CENTRE NUMBER			CANDIDATE NUMBER		

FURTHER MATHEMATICS

9231/43

Paper 4 Further Probability & Statistics

October/November 2021

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer all questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do not write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

This document has 16 pages. Any blank pages are indicated.

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	$\overline{x} = 25.6 \qquad \qquad \sum (x - \overline{x})^2 = 78.5$	
(a)	Find a 90% confidence interval for μ .	[4]
A te	est of the null hypothesis $\mu=k$ is carried out on this sample, using a 10 does not support the alternative hypothesis $\mu < k$.	0% significance level. The
test	est of the null hypothesis $\mu = k$ is carried out on this sample, using a 10 does not support the alternative hypothesis $\mu < k$. Find the greatest possible value of k .	0% significance level. The
test	t does not support the alternative hypothesis $\mu < k$.	
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2	The continuous	random	variable X has	s cumulative	distribution	function F	given	hν

$$F(x) = \begin{cases} 0 & x < -1, \\ \frac{1}{2}(1+x)^2 & -1 \le x \le 0, \\ 1 - \frac{1}{2}(1-x)^2 & 0 < x \le 1, \\ 1 & x > 1. \end{cases}$$

(a)	Find the probability density function of X .	[2]
(b)	Find $P\left(-\frac{1}{2} \leqslant X \leqslant \frac{1}{2}\right)$.	[2]

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Find $Var(X^2)$.	

3 A supermarket sells pears in packs of 8. Some of the pears in a pack may not be ripe, and the supermarket manager claims that the number of unripe pears in a pack can be modelled by the distribution B(8, 0.15).

A random sample of 150 packs was selected and the number of unripe pears in each pack was recorded. The following table shows the observed frequencies together with some of the expected frequencies using the manager's binomial distribution.

Number of unripe pears per pack	0	1	2	3	4	5	≥6
Observed frequency	35	48	43	15	6	3	0
Expected frequency	40.874	p	35.641	12.579	2.775	0.392	q

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4 Manet has developed a new training course to help athletes improve their time taken to run 800 m. Manet claims that his course will decrease an athlete's time by more than 2 s on average. For a random sample of 10 athletes the times taken, in seconds, before and after the course are given in the following table.

Athlete	A	В	С	D	E	F	G	Н	I	J
Before	150	146	131	135	126	142	130	129	137	134
After	145	138	129	135	122	135	132	128	127	137

Use a <i>t</i> -test, at the 5% assumption that you ma	% significance ke.	level, to test	whether Ma	anet's claim is	s justified,	stating any [8]
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5

(a)	Find the probability generating function $G_X(t)$ of X .	[3
he	e balls are replaced in the bag.	
acc	e balls are replaced in the bag. ob now selects two balls at random from the bag, without replacement. The randon on the balls selected by Jacob that are labelled with an even number.	n variable Y is th
Jaco nun	ob now selects two balls at random from the bag, without replacement. The randon	n variable Y is the $[2]$
acc	ob now selects two balls at random from the bag, without replacement. The randon nber of balls selected by Jacob that are labelled with an even number.	
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lacc	ob now selects two balls at random from the bag, without replacement. The randon nber of balls selected by Jacob that are labelled with an even number.	
Jaco num	ob now selects two balls at random from the bag, without replacement. The randon nber of balls selected by Jacob that are labelled with an even number.	

The random variable *Z* is the sum of the number of balls that are labelled with a multiple of 3 selected by Kai and the number of balls that are labelled with an even number selected by Jacob.

Find the pro				
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Use the prob	ability generati	ng function of Z t	so find $\mathrm{E}(Z)$.	
Use the prob	ability generati	ng function of Z t	o find E(Z).	

Women	51	55	242	167	152	256	75	137	98	238	235
Men	311	262	170	302	175	320	220	260	72	351	86
Carry out a	Wilcox	on rank-	sum tes	t, at the	5% sig	nificanc	e level,	to test w	hether,	on aver	age, the
difference i	n choles	terol lev	vels bety	ween we	omen ar	nd men.					
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Additional Page

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