



Cambridge International AS & A Level

CANDIDATE
NAME

--

CENTRE
NUMBER

--	--	--	--	--

CANDIDATE
NUMBER

--	--	--	--

MATHEMATICS

9709/33

Paper 3 Pure Mathematics 3

May/June 2021

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.



- 1** Expand $(1 + 3x)^{\frac{2}{3}}$ in ascending powers of x , up to and including the term in x^3 , simplifying the coefficients. [4]

This image shows a full page of a worksheet designed for handwriting practice. It consists of approximately 20 horizontal dashed lines spaced evenly across the page, providing a guide for letter height and placement. The background is plain white, and there are no other markings or text present.

- 2** Solve the equation $4^x = 3 + 4^{-x}$. Give your answer correct to 3 decimal places. [5]

[illegible]

- 3 The parametric equations of a curve are

$$x = t + \ln(t + 2), \quad y = (t - 1)e^{-2t},$$

where $t > -2$.

- (a) Express $\frac{dy}{dx}$ in terms of t , simplifying your answer. [5]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (b) Find the exact y -coordinate of the stationary point of the curve. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

4 Let $f(x) = \frac{15 - 6x}{(1 + 2x)(4 - x)}$.

(a) Express $f(x)$ in partial fractions.

[3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(b) Hence find $\int_1^2 f(x) \, dx$, giving your answer in the form $\ln\left(\frac{a}{b}\right)$, where a and b are integers. [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- 5 (a)** By first expanding $\tan(2\theta + 2\theta)$, show that the equation $\tan 4\theta = \frac{1}{2} \tan \theta$ may be expressed as $\tan^4 \theta + 2 \tan^2 \theta - 7 = 0$. [4]

This image shows a full page of a worksheet designed for handwriting practice. It consists of approximately 20 horizontal rows. Each row is defined by two parallel dotted lines, creating a series of uniform gaps where letters can be written. The lines are evenly spaced across the entire page, providing a guide for letter height and placement. There is no text or other markings on the page.

© UCLES 2021

- 6 (a) By sketching a suitable pair of graphs, show that the equation $\cot \frac{1}{2}x = 1 + e^{-x}$ has exactly one root in the interval $0 < x \leq \pi$. [2]

- (b) Verify by calculation that this root lies between 1 and 1.5. [2]

.....

.....

.....

.....

.....

.....

.....

.....

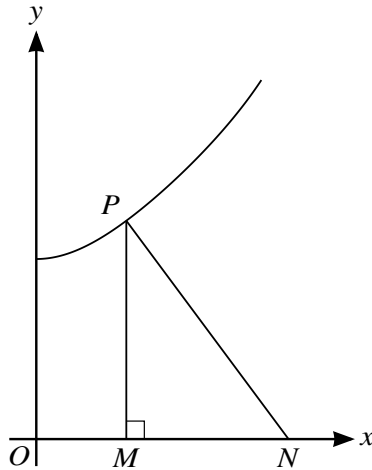
.....

.....

.....

This image shows a full page of a worksheet designed for handwriting practice. It consists of approximately 20 horizontal rows. Each row is defined by two parallel dotted lines, creating a series of uniform gaps for writing. The lines are evenly spaced across the entire page, providing a guide for letter height and placement. There is no text or other markings on the page.

7



For the curve shown in the diagram, the normal to the curve at the point P with coordinates (x, y) meets the x -axis at N . The point M is the foot of the perpendicular from P to the x -axis.

The curve is such that for all values of x in the interval $0 \leq x < \frac{1}{2}\pi$, the area of triangle PMN is equal to $\tan x$.

- (a) (i) Show that $\frac{MN}{y} = \frac{dy}{dx}$. [1]

.....

.....

.....

.....

.....

- (ii) Hence show that x and y satisfy the differential equation $\frac{1}{2}y^2 \frac{dy}{dx} = \tan x$. [2]

.....

.....

.....

.....

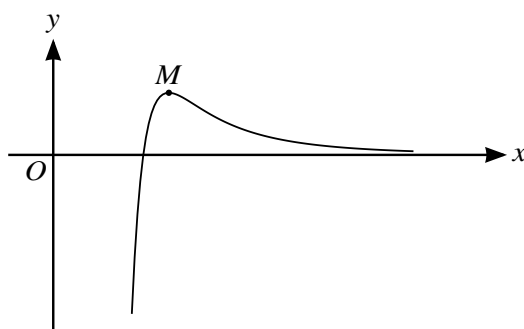
.....

.....

.....

- (b) Given that $y = 1$ when $x = 0$, solve this differential equation to find the equation of the curve, expressing y in terms of x . [6]

[illegible]



(a) Find the exact coordinates of M . [4]

This image shows a full page of white paper with horizontal dotted lines, resembling notebook paper. The lines are evenly spaced and run across the width of the page. There is no handwriting or other markings on the paper.

(b) By using integration by parts, show that for all $a > 1$, $\int_1^a \frac{\ln x}{x^4} dx < \frac{1}{9}$. [6]

[illegible]

- 9 The quadrilateral $ABCD$ is a trapezium in which AB and DC are parallel. With respect to the origin O , the position vectors of A , B and C are given by $\vec{OA} = -\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$, $\vec{OB} = \mathbf{i} + 3\mathbf{j} + \mathbf{k}$ and $\vec{OC} = 2\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$.

- (a) Given that $\overrightarrow{DC} = 3\overrightarrow{AB}$, find the position vector of D . [3]

This image shows a full page of white paper with horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- (b) State a vector equation for the line through A and B . [1]

[illegible]

This image shows a full page of white paper with horizontal dotted lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- 10 (a) Verify that $-1 + \sqrt{2}i$ is a root of the equation $z^4 + 3z^2 + 2z + 12 = 0$. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (b) Find the other roots of this equation. [7]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

[illegible]

BLANK PAGE

Permission to reproduce items where third-party owned material protected by copyright is included has been sought and cleared where possible. Every reasonable effort has been made by the publisher (UCLES) to trace copyright holders, but if any items requiring clearance have unwittingly been included, the publisher will be pleased to make amends at the earliest possible opportunity.

To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced online in the Cambridge Assessment International Education Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download at www.cambridgeinternational.org after the live examination series.

Cambridge Assessment International Education is part of the Cambridge Assessment Group. Cambridge Assessment is the brand name of the University of Cambridge Local Examinations Syndicate (UCLES), which itself is a department of the University of Cambridge.