

# Cambridge International AS & A Level

CANDIDATE NAME						
CENTRE NUMBER				CANDIDATE NUMBER		

# 246420934

#### **FURTHER MATHEMATICS**

9231/23

Paper 2 Further Pure Mathematics 2

May/June 2022

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

#### **INSTRUCTIONS**

- Answer all questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do not use an erasable pen or correction fluid.
- Do not write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

#### **INFORMATION**

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has 20 pages. Any blank pages are indicated.

Find the roots of the equatio $-\pi \le \theta < \pi$ .		[5
	 	 ••••••
	 	 •••••
	 	 •••••

()	Find the coefficient of $x^2$ in the Maclaurin's series for $-\ln \cos x$ .	
<b>(b)</b>	Find the length of the are of the curve with equation $y = -\ln \cos x$ from the point	nt where x =
	Find the length of the arc of the curve with equation $y = -\ln \cos x$ from the point where $x = \frac{1}{4}\pi$ .	nt where $x =$
	Find the length of the arc of the curve with equation $y = -\ln \cos x$ from the point where $x = \frac{1}{4}\pi$ .	nt where  x =
	the point where $x = \frac{1}{4}\pi$ .	
	the point where $x = \frac{1}{4}\pi$ .	
	the point where $x = \frac{1}{4}\pi$ .	
	the point where $x = \frac{1}{4}\pi$ .	
	the point where $x = \frac{1}{4}\pi$ .	
	the point where $x = \frac{1}{4}\pi$ .	
	the point where $x = \frac{1}{4}\pi$ .	
	the point where $x = \frac{1}{4}\pi$ .	
	the point where $x = \frac{1}{4}\pi$ .	
	the point where $x = \frac{1}{4}\pi$ .	

3 The matrix A is given by

$$\mathbf{A} = \begin{pmatrix} 6 & -9 & 5 \\ 5 & -8 & 5 \\ 1 & -1 & 2 \end{pmatrix}.$$

•••••

Use the charace be determined.	teristic equation	of <b>A</b> to show the	hat $\mathbf{A}^{-1} = p\mathbf{A}^2$	$+q\mathbf{I}$ , where $p$ as	nd q are constant	ts to [3]
•••••						
•••••						

4	T		. 1	
4	- I f 14	s giv	en f	hat
-	1111	<i>3</i> <b>5</b> 1 <b>7</b>	OII U	mu

		$x = -t + \tan^{-1} t$	and	$y = t + \sinh^{-1} t.$	
(a)	Show that $\frac{dy}{dx} = -\frac{t^2}{2}$	$\frac{t^2 + 1 + \sqrt{t^2 + 1}}{t^2}.$			[4]

ľ	ind the value of $\frac{d^2y}{dx^2}$ when $t = \frac{3}{4}$ .
•	

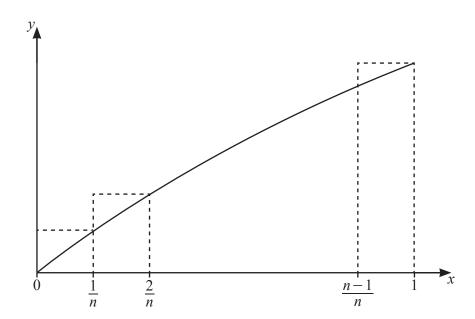
5 Find the solution of the differential equation

$x(x+7)\frac{\mathrm{d}y}{\mathrm{d}x} + 7y$	, =	x
$dx \cdot dx$		

for which $y = 7$ when $x = 1$ . Give your answer in the form $y = f(x)$ .	[9]

•••••
•••••
•••••
•••••

6



The diagram shows the curve with equation  $y = \ln(1+x)$  for  $0 \le x \le 1$ , together with a set of n rectangles each of width  $\frac{1}{n}$ .

(a)	By considering the sum of the areas of these rectangles, show that	$\int_0^1 \ln(1+x)  \mathrm{d}x < U_n, \text{ where}$	;
	$U_n = \frac{1}{n} \ln \frac{(2n)!}{n!} - \ln n.$		[4]

					n	For $\int_0^1 \ln(1 - 1)$	,	
							•••••	
							•••••	
							•••••	
By simpl	ifying $U_n$ –	$-L_{}$ , show t	that $\lim (U)$	$(L_{n}-L_{n})=0$	).			
by simpi	n	$L_n$ , show t	$n \to \infty$	n = n	<i>.</i>			
•••••				••••••	•••••			
•••••								

7	The variables $x$ a	ind y are	related by	the differential	equation
---	---------------------	-----------	------------	------------------	----------

erential equation
$$4\frac{d^2y}{dx^2} - y = 3.$$

It is given that, when x = 0, y = -3 and  $\frac{dy}{dx} = 2$ .

Find $y$ in terms of $x$ .	

• • • • • • • • • • • • • • • • • • • •					
•••••					
•••••					
					•••••
•••••		•••••	•••••		
educe the exact	value of x for which	ch y = 0. Give yo	ur answer in loga	rithmic form.	[3]
educe the exact	value of x for which	ch y = 0. Give yo	ur answer in loga	rithmic form.	[3]
educe the exact	value of x for which	ch y = 0. Give yo	ur answer in loga	rithmic form.	[3]
duce the exact	value of x for which	ch y = 0. Give yo	ur answer in loga	rithmic form.	[3]
luce the exact	value of x for which	ch y = 0. Give yo	ur answer in loga	rithmic form.	[3]
educe the exact	value of x for which	ch y = 0. Give yo	ur answer in loga	rithmic form.	[3]
duce the exact	value of x for which	ch $y = 0$ . Give yo	ur answer in loga	rithmic form.	[3]
duce the exact	value of x for which	ch $y = 0$ . Give yo	ur answer in loga	rithmic form.	[3]
duce the exact	value of x for which	ch $y = 0$ . Give yo	ur answer in loga	rithmic form.	[3]
uce the exact	value of x for which	ch $y = 0$ . Give yo	ur answer in loga	rithmic form.	[3]
duce the exact	value of x for which	ch $y = 0$ . Give yo	ur answer in loga	rithmic form.	[3]
uce the exact	value of x for which	ch $y = 0$ . Give yo	ur answer in loga	rithmic form.	[3]
luce the exact	value of x for which	ch <i>y</i> = 0. Give yo	ur answer in loga		[3]
duce the exact	value of x for which	ch $y = 0$ . Give yo	ur answer in loga		
educe the exact	value of x for which	ch $y = 0$ . Give yo	ur answer in loga		
Deduce the exact	value of x for which	ch $y = 0$ . Give yo	ur answer in loga		

	$I_{m,n} = \int_0^{\frac{1}{2}\pi} \sin^m \theta \cos^n \theta  \mathrm{d}\theta.$	
(b)	Show that, for $m \ge 2$ and $n \ge 0$ , $m-1$	
	$I_{m,n} = \frac{m-1}{m+n} I_{m-2,n}.$	

	$\cos^5\theta = a\cos 5\theta + b\cos 3\theta + c\cos \theta,$	
where $a$ , $b$ and $c$ are $c$	onstants to be determined.	

Using the results given in parts (b) and (c), find the exact value of $I_{2,5}$ .	[4]

# Additional page

If you use the following page to complete the answer to any question, the question number must shown.	be clearly
	,
	•••••

## **BLANK PAGE**

## **BLANK PAGE**

#### **BLANK PAGE**

Permission to reproduce items where third-party owned material protected by copyright is included has been sought and cleared where possible. Every reasonable effort has been made by the publisher (UCLES) to trace copyright holders, but if any items requiring clearance have unwittingly been included, the publisher will be pleased to make amends at the earliest possible opportunity.

To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced online in the Cambridge Assessment International Education Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download at www.cambridgeinternational.org after the live examination series.

Cambridge Assessment International Education is part of Cambridge Assessment. Cambridge Assessment is the brand name of the University of Cambridge Local Examinations Syndicate (UCLES), which is a department of the University of Cambridge.