

Cambridge International AS & A Level

CANDIDATE NAME					
CENTRE NUMBER			CANDIDATE NUMBER		

7 3 2 7 5 4 3 3 3

FURTHER MATHEMATICS

9231/33

Paper 3 Further Mechanics

May/June 2022

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer all questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do not use an erasable pen or correction fluid.
- Do not write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity (g) is needed, use $10 \,\mathrm{m\,s^{-2}}$.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

This document has 16 pages. Any blank pages are indicated.

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Find the x -coordinate of the centre of mass of the lamina.	[4]
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A particle P of mass m is attached to one end of a light elastic string of natural length a and modulus of elasticity $\frac{4}{3}mg$. The other end of the string is attached to a fixed point O on a rough horizontal surface. The particle is at rest on the surface with the string at its natural length. The coefficient of friction between P and the surface is $\frac{1}{3}$. The particle is projected along the surface in the direction OP with a speed of $\frac{1}{2}\sqrt{ga}$.				
Find the greatest extension of the string during the subsequent motion. [5]				

a)	Find the value of $\sin \theta$.	
		•••••

Find the range of the flight.	[3
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One end of a light inextensible string of length a is attached to a fixed point O. A particle of mass m is attached to the other end of the string and is held with the string taut at the point A. At A the string makes an angle θ with the upward vertical through O. The particle is projected perpendicular to the string in a downward direction from A with a speed u. It moves along a circular path in the vertical plane.

When the string makes an angle α with the downward vertical through O, the speed of the particle is 2u and the magnitude of the tension in the string is 10 times its magnitude at A.

It is given that $u = \sqrt{\frac{2}{3}ga}$.

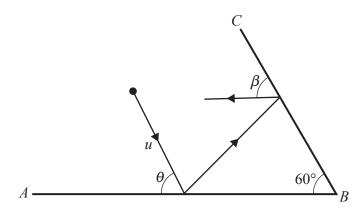
(a)	Find, in terms of m and g , the magnitude of the tension in the string at A .	[6]
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						b)	Find the value of $\cos \alpha$.	[2]
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	tive force of magnitude $(4e^{-x} + 12)e^{-x}N$. When $t = 0$, $x = 0$ and $v = 4$.	
a)	Show by integration that $v = \frac{1+3e^x}{e^x}$.	[4

6

(a)

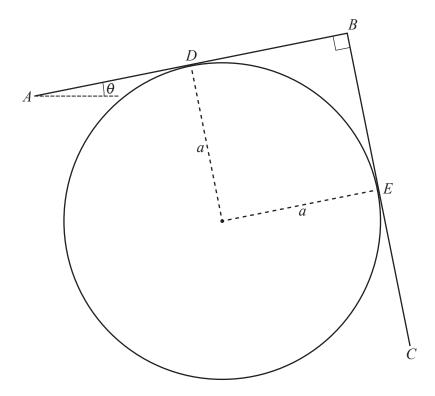


AB and BC are two fixed smooth vertical barriers on a smooth horizontal surface, with angle $ABC = 60^{\circ}$. A particle of mass m is moving with speed u on the surface. The particle strikes AB at an angle θ with AB. It then strikes BC and rebounds at an angle β with BC (see diagram). The coefficient of restitution between the particle and each barrier is e and $\tan \theta = 2$.

The kinetic energy of the particle after the first collision is 40% of its kinetic energy before the first collision.

Find the value of <i>e</i> .	[4]

)	Find the size of angle β .	[4]
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A uniform cylinder with a rough surface and of radius a is fixed with its axis horizontal. Two identical uniform rods AB and BC, each of weight W and length 2a, are rigidly joined at B with AB perpendicular to BC. The rods rest on the cylinder in a vertical plane perpendicular to the axis of the cylinder with AB at an angle θ to the horizontal. D and E are the midpoints of E and E respectively and also the points of contact of the rods with the cylinder (see diagram). The rods are about to slip in a clockwise direction. The coefficient of friction between each rod and the cylinder is μ .

The normal reaction between AB and the cylinder is R and the normal reaction between BC and the cylinder is N.

) Find the ratio $R: N$ in terms of μ .	[6]

(h)	Given that $\mu = \frac{1}{3}$, find the value of $\tan \theta$. [3]
(0)	$\frac{1}{3}, \text{ find the value of tail o}.$

Additional page

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