

Cambridge International AS & A Level

CANDIDATE NAME					
CENTRE NUMBER			CANDIDATE NUMBER		

0510110257

FURTHER MATHEMATICS

9231/22

Paper 2 Further Pure Mathematics 2

October/November 2022

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer all questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do not use an erasable pen or correction fluid.
- Do not write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has 16 pages.

	Find the set of values of k for which the system of equations $x + 2y + 3z = 1,$	
	kx + 4y + 6z = 0,	
	7x + 8y + 9z = 3,	
	has a unique solution.	[3
(b)	Interpret the situation geometrically in the case where the system of equation unique solution.	
	unique solution.	[2

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		(x +	$1)y + y^2 = 2.$	
(a)	Show that $\frac{dy}{dx} = -$	$-\frac{2}{3}$ at the point $(0, -2)$).	[3]
		d^2v		
(b)	Find the value of	$\frac{d^2y}{dx^2}$ at the point $(0, -\frac{1}{2})$		 [4]

sur	face generated when the curve is rotated through 2π radians about the x-axis.	
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		, or otherwise,	find the Macla	urin's series [2]

4 Find the solution of the differential equation

$(4t^2 -$	$1)\frac{\mathrm{d}x}{\mathrm{d}t}$	+4x	=	$4t^2$	_	1
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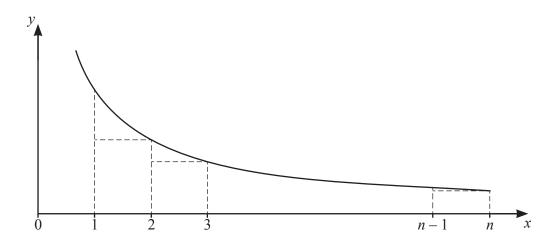
for which $x = 3$ when $t = 1$. Give your answer in the form $x = f(t)$.	[9]
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(a)	Write down the fourth roots of unity.	[1
(b)	Use de Moivre's theorem to show that	
	$\cos 4\theta = 8\cos^4\theta - 8\cos^2\theta + 1.$	[4
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16	$(8x^4 -$	$-8r^2$	+1	⁴ –	9 =	- 0
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in the form $cos(q\pi)$, where q is a rational number.	[5]

6



The diagram shows the curve $y = \frac{1}{\sqrt{x^2 + 2x}}$ for x > 0, together with a set of (n-1) rectangles of unit width.

By considering the sum of the areas of these rectangles, show that

$\sum_{r=1}^{n} \frac{1}{\sqrt{r^2 + 2r}} < \ln(n + 1 + \sqrt{n^2 + 2n}) + \frac{1}{3}\sqrt{3} - \ln(2 + \sqrt{3}).$	10]
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' (a) It is given that λ is an eigenvalue of the non-singular square matrix \mathbf{A} , with corresponding eigenvector \mathbf{e} .
	Show that λ^{-1} is an eigenvalue of \mathbf{A}^{-1} for which \mathbf{e} is a corresponding eigenvector. [2]
T	he matrix A is given by
	$\mathbf{A} = \begin{pmatrix} 2 & 0 & 3 \\ 15 & -4 & 3 \\ 3 & 0 & 2 \end{pmatrix}.$
(b	Given that -1 is an eigenvalue of \mathbf{A} , find a corresponding eigenvector. [2]
(c) It is also given that $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$ are eigenvectors of A . Find the corresponding eigenvalues. [2]

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Use the ch	aracteristic	e equation	of A to	show that	$\mathbf{A}^{-1} = n$	$A^2 + aI$	where n	and q are ra
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8 It is given that $y = \cosh u$, where u > 0, and

$$\sqrt{\cosh^2 u - 1} \left(\frac{\mathrm{d}^2 u}{\mathrm{d}x^2} + \frac{\mathrm{d}u}{\mathrm{d}x} \right) + \cosh u \left(\frac{\mathrm{d}u}{\mathrm{d}x} \right)^2 - 2\cosh u = 4\mathrm{e}^{-x}.$$

(a) Show that

(b)

	$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + \frac{\mathrm{d}y}{\mathrm{d}x} - 2y = 4\mathrm{e}^{-x}.$	[4]
Find u in terms of x , given that,	when $x = 0$, $u = \ln 3$ and $\frac{du}{dx} = 3$.	[10]

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Additional page

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