Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

9895853091

FURTHER MATHEMATICS **9231/43**

Paper 4 Further Probability & Statistics **October/November 2022**

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS
- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION
- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

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1 Jasmine is researching the heights of pine trees in forests in two regions A and B. She chooses a random sample of 50 pine trees in region A and records their heights, x m. She also chooses a random sample of 60 pine trees in region B and records their heights, y m. Her results are summarised as follows.

$$\sum x = 1625 \qquad \sum x^2 = 53200 \qquad \sum y = 1854 \qquad \sum y^2 = 57900$$

Find a 95% confidence interval for the difference between the population mean heights of pine trees in regions A and B. [7]

2 An organisation runs courses to train students to become engineers. These students are taught in groups of 8. The director of the organisation claims that on average 60% of the students in a group achieve a pass. A random sample of 150 groups of 8 students is chosen. The following table shows the observed frequencies together with some of the expected frequencies using the appropriate binomial distribution.

Number of passes per group	0	1	2	3	4	5	6	7	8
Observed frequency	0	0	8	24	45	36	26	10	1
Expected frequency	p	1.180	6.193	18.579	34.836	q	r	13.437	2.519

(a) Find the values of p, q and r giving your answers correct to 3 decimal places. [2]

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(b) Carry out a goodness of fit test, at the 10% significance level, to test whether there is evidence to reject the director's claim. [6]

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3 A large college is holding a piano competition. Each student has played a particular piece of music and two judges have each awarded a mark out of 80. The marks awarded to a random sample of 14 students are shown in the following table.

Student	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*	*J*	*K*	*L*	*M*	*N*
Judge 1	79	54	63	74	69	52	50	57	55	42	63	55	56	48
Judge 2	75	62	60	73	76	41	31	51	45	55	49	50	65	36

(a) One of the students claims that on average Judge 1 is awarding higher marks than Judge 2. Carry out a Wilcoxon matched-pairs signed-rank test at the 5% significance level to test whether the data supports the student's claim. [7]

(b) Give a reason why it is preferable to use a Wilcoxon matched-pairs signed-rank test in this situation rather than a paired sample t-test. [1]

4 Jason has three biased coins. For each coin the probability of obtaining a head when it is thrown is $\frac{2}{3}$. Jason throws all three coins. The number of heads obtained is denoted by X.

(a) Find the probability generating function $G_X(t)$ of X. [3]

Jason also has two unbiased coins. He throws all five coins. The number of heads obtained from the two unbiased coins is denoted by Y. It is given that $G_Y(t) = \frac{1}{4} + \frac{1}{2}t + \frac{1}{4}t^2$. The random variable Z is the total number of heads obtained when Jason throws all five coins.

(b) Find the probability generating function of Z, expressing your answer as a polynomial. [3]

(c) Find E(Z). [2]

5 The continuous random variable X has cumulative distribution function F given by

$$\mathrm{F}(x) = \begin{cases} 0 & x < 0, \\ 1 - \frac{1}{144}(12 - x)^2 & 0 \leqslant x \leqslant 12, \\ 1 & x > 12. \end{cases}$$

(a) Find the upper quartile of X. [2]

(b) Find $\mathrm{Var}(X^2)$. [5]

The random variable Y is given by $Y = \sqrt{X}$.

(c) Find the probability density function of Y. [3]

6 A company manufactures copper pipes. The pipes are produced by two different machines, A and B. An inspector claims that the mean diameter of the pipes produced by machine A is greater than the mean diameter of the pipes produced by machine B. He takes a random sample of 12 pipes produced by machine A and measures their diameters, x cm. His results are summarised as follows.

$$\sum x = 6.24 \qquad \sum x^2 = 3.26$$

He also takes a random sample of 10 pipes produced by machine B and measures their diameters in cm. His results are as follows.

0.48	0.53	0.47	0.54	0.54	0.55	0.46	0.55	0.50	0.48

The diameters of the pipes produced by each machine are assumed to be normally distributed with equal population variances.

Test at the 2.5% significance level whether the data supports the inspector's claim. [9]

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Additional page

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